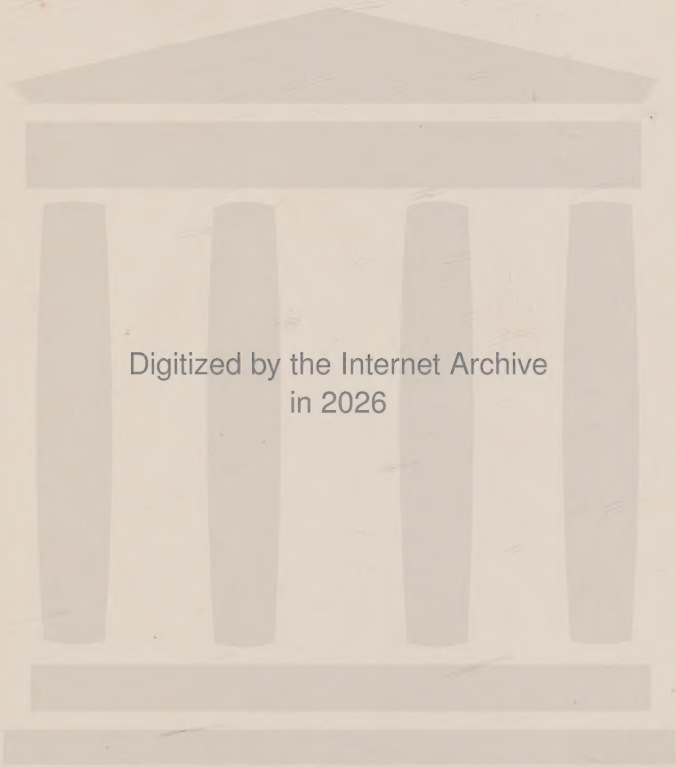
PRACTICAL MATHEMATICS

PART IV

AMERICAN SCHOOL OF CORRESPONDENCE
CHICAGO, ILLINOIS



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PRACTICAL MATHEMATICS

PART IV

INSTRUCTION PAPER

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PRACTICAL MATHEMATICS

PART IV

PRACTICAL GEOMETRY

Geometry, the science of space, treats of lines, triangles, etc.

A figure of one dimension, *length* is called a *line;* a figure of two dimensions, *length* and *breadth,* is called a *surface* or *area;* for example, a triangle, a circle; a figure of three dimensions, *length, breadth,* and *thickness,* is called a *solid;* for example, a cube, a sphere, a cone. Such figures are called *geometrical figures.*

LINES

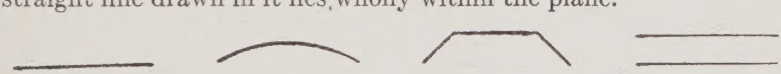
74. A *line* has length only; it is made by the motion of a point.

A *straight line* is one that has the same direction throughout. It is the shortest distance between two points.

A *curved line* is one that is constantly changing in direction; it is sometimes called a *curve.*

A *broken line* is one made up of several straight lines.

A *plane surface* (or simply a *plane*) is a surface such that any straight line drawn in it lies wholly within the plane.



Straight Line. Curved Line. Broken Line. Parallel Lines.

Parallel lines are lines which lie in the same plane and are equally distant from each other at all points.

Lines are lettered to distinguish them; thus if one end is marked *a* and the other *b*, it is called "the line *ab*" or "the line *ba.*"

ANGLES

75. An *angle* is the measure of the difference in direction of two lines. The lines are called *sides,* and the point of meeting, the *vertex.* The size of an angle is independent of the length of its sides.

If one straight line meets another (not at an extremity), Fig. 9, so the two angles formed are equal, the lines are said to be *perpendicular* to each other and the angles formed are called *right angles.*

An *acute angle* is less than a right angle, Fig. 10.

An *obtuse angle* is greater than a right angle, Fig. 11.

Angles are distinguished by placing letters at the end of the lines forming the sides and at the vertex. For example, in Fig. 10 the angle would be read as "the angle $A\ B\ C$" or "the angle $C\ B\ A$." In writing, the sign $\angle$ is often used in place of the word "angle."

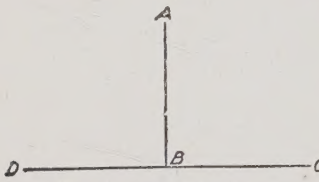


Fig. 9. Right Angle.

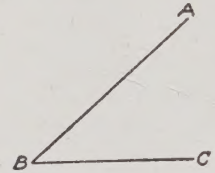


Fig. 10. Acute Angle.

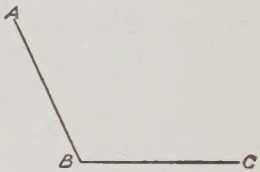


Fig. 11. Obtuse Angle.

As two right angles are formed when one line meets another perpendicularly, Fig. 9, it follows that:

(a) *The sum of all the angles about a point on one side of a straight line is equal to two right angles.*

(b) *The sum of all the angles about any point is equal to four right angles.*

When two lines intersect they form four angles, Fig. 12. The angles $A\ O\ C$ and $A\ O\ D$ are called *adjacent* angles because they have the same vertex O and the line $A\ O$ is common to both as it forms a side of each angle. It has been proved that $\angle A\ O\ C + \angle A\ O\ D =$ two right angles and it may also be proved that the angles $A\ O\ C$ and $D\ O\ B$, called *opposite* angles, are equal to each other. Therefore, *when two straight lines intersect, the opposite angles formed are equal to each other.*

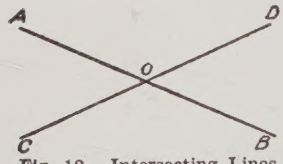


Fig. 12. Intersecting Lines.

76. Measurement of Angles. It has been shown in Fig. 12, that the sum of the angles about the point of intersection of two lines is always four right angles. Evidently, other lines might be drawn through the same intersection, making the angles smaller and more in number, without changing the value of their sum. In measuring angles, therefore, it has been agreed to divide these four right angles about a point into 360 equal parts called *degrees*, indicated by the sign, °. In this case each of the four right angles would contain

90° while one-half, two-thirds, and one-third of a right angle would represent angles of respectively 45°, 60°, and 30°, as shown in Fig. 13. In Drawing Work, it will be found that the 45° triangle and the 30° — 60° triangle are useful articles of equipment, but when it comes to accurate values of angles, some device like a *protractor*, Fig. 14, showing degrees and fractions of degrees, is necessary. Each degree is divided into 60 equal parts called *minutes*, ′, and each minute into 60 equal parts called *seconds*, ″. Thus to represent an angle of 23 degrees, 47 minutes, and 9 seconds, write 23° 47′ 9″.

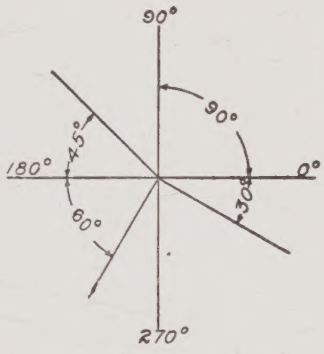


Fig. 13.

The relations of these units may be summarized as follows:

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circle

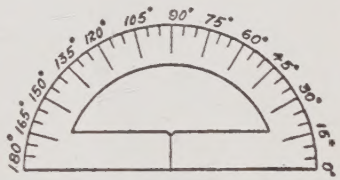


Fig. 14. Protractor.

Two angles are *complementary* when their sum is equal to one right angle or 90°; they are *supplementary* when their sum is equal to two right angles or 180°. Thus, an angle of 33° is the complement of one of 57° because 33 + 57 = 90. Angles of 148° and 32° are supplementary because 148 + 32 = 180.

PROBLEMS FOR PRACTICE

1. How many seconds in 180 degrees? Ans. 648,000
2. How many right angles in 202° 30′? Ans. $2\frac{1}{4}$
3. What is the complement of 37°?
4. What is the complement of 11° 47′ 3″? Ans. 78° 12′ 57″
5. What is the supplement of 131° 4′ 27″? Ans. 48° 55′ 33″
6. Does an acute angle contain more or less than 90°? An obtuse angle?
7. In Fig. 12, the angle $A\ O\ C$ is, say, 52°; find the value of the three other angles about the point O.
8. How many seconds in 30′?

POLYGONS

77. A *polygon* is a plane figure bounded by straight lines. The boundary lines are called the *sides* and the sum of the sides is called the *perimeter*.

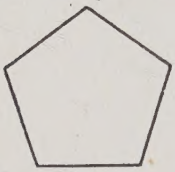


Fig. 15. Pentagon.

Fig. 16. Hexagon.

Fig. 17. Octagon.

Polygons are classified according to the number of sides.

A *triangle* is a polygon of three sides.

A *quadrilateral* is a polygon of four sides.

A *pentagon* is a polygon of five sides, Fig. 15.

A *hexagon* is a polygon of six sides, Fig. 16.

An *octagon* is a polygon of eight sides, Fig. 17.

An *equilateral polygon* is one all of whose sides are equal.

An *equiangular polygon* is one all of whose angles are equal.

A *regular polygon* is one all of whose angles and all of whose sides are equal.

TRIANGLES

78. A *triangle* is a polygon enclosed by three straight lines called *sides*. The *angles* of a triangle are the angles formed by the sides.

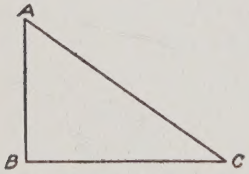


Fig. 18. Right-Angled Triangle.

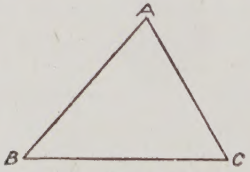


Fig. 19. Acute-Angled Triangle.

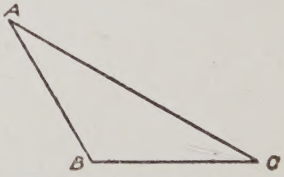


Fig. 20. Obtuse-Angled Triangle.

A *right-angled* triangle, often called a *right* triangle, Fig. 18, is one that has a right angle. The longest side (the one opposite the right angle) is called the *hypotenuse* and the other sides are sometimes called *legs*.

An *acute-angled* triangle, Fig. 19, is one that has all of its angles acute.

An *obtuse-angled* triangle, Fig. 20, is one that has an obtuse angle.

An *equilateral* triangle, Fig. 21, is one having all of its sides equal.

An *equiangular* triangle is one having all of its angles equal.

An *isosceles* triangle, Fig. 22, is one two of whose sides are equal.

A *scalene* triangle, Fig. 23, is one no two of whose sides are equal.

(a) The *base* of a triangle is the lowest side; it is the side upon which the triangle is supposed to stand. Any side may, however, be taken as the base. In an isosceles triangle, the side which is not one of the equal sides is usually considered as the base.

Fig. 21.
Equilateral Triangle.

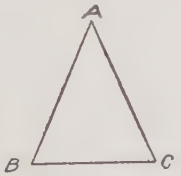


Fig. 22.
Isosceles Triangle.

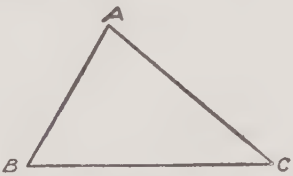


Fig. 23.
Scalene Triangle.

(b) The *altitude* of a triangle is the perpendicular drawn from the vertex to the base *A D*, Fig. 24. In some triangles, Fig. 25, it is

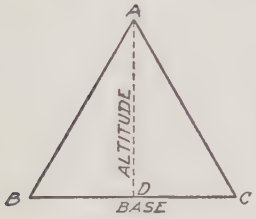


Fig. 24.

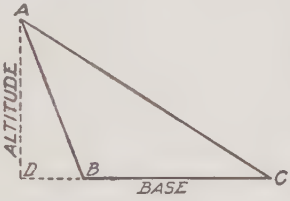


Fig. 25.

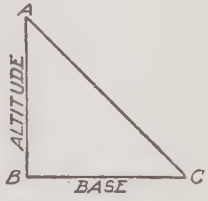


Fig. 26.

necessary to produce the base so that the altitude may meet it. In a right triangle one leg may be considered as the base and the other as the altitude, Fig. 26.

Rule: *The sum of the angles of any triangle is equal to two right angles.* This simple relation may be proved as follows: Let the base *B C* of the triangle in Fig. 27 be extended or produced to *E*, and the line *C D* be drawn parallel to *A B*. This construction makes the $\angle D C E$ equal to the $\angle A B C$, and the $\angle B A C$ equal to the $\angle ACD$. Forming an equation from these two equalities and adding the $\angle A C B$ to both sides, the following is obtained:

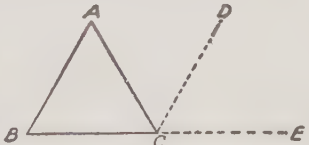


Fig. 27.

$$A B C + B A C + A C B = D C E + A C D + A C B$$

But it will be noticed that the left member of this equation comprises the three angles of the triangle $A\,B\,C$ and that the right member includes all of the angles about the point C on one side of the line $B\,E$, which have already been found to equal two right angles.

Hence, if

$$D\,C\,E + A\,C\,D + A\,C\,B = \text{two right angles}$$

then

$$A\,B\,C + B\,A\,C + A\,C\,B = \text{two right angles}$$

Therefore the sum of *all* the angles of any triangle is equal to two right angles.

(c) *Law of the right triangle.* If in the right triangle $A\,B\,C$, Fig. 28, the side $A\,B$ is made 3 inches long, and the side $A\,C$ is made four inches long, then the side $B\,C$, called the *hypotenuse*, is found to be 5 inches long.

The proof of this may be found by considering the accompanying figure. The square $A\,B\,E\,D$ constructed on the side $A\,B$ contains

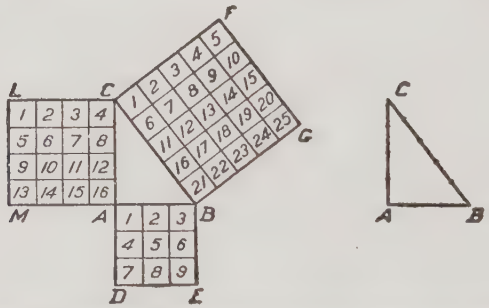


Fig. 28. Graphical Proof of the Law of the Right Triangle.

9 squares; the square $L\,C\,A\,M$ constructed on the side $A\,C$ contains 16 squares, and the square $C\,F\,G\,B$ constructed on the hypotenuse $B\,C$ contains 25 squares. Now,

$$9 + 16 = 25$$

$$3^2 + 4^2 = 5^2$$

$$\overline{AB}^2 + \overline{AC}^2 = \overline{BC}^2$$

Rule: *The square of the hypotenuse is equal to the sum of the squares of the other two sides.* This statement is only true for *right triangles*, and hence must not be applied to triangles in general. By means of this relation it is possible to find the length of one side of a right triangle if the other two are known.

Examples. 1. A right triangle has sides 6 inches and 10 inches long respectively; what is the length of the hypotenuse?

SOLUTION. Let x represent the hypotenuse.

$$x^2 = 6^2 + 10^2$$
$$= 36 + 100 = 136$$
$$x = \sqrt{136} = 11.66 + \text{ inches.} \quad \text{Ans.}$$

2. The hypotenuse of a right triangle is 24 feet long and one of the short sides is 9 feet. How long is the other side?

SOLUTION. Let x represent the other side.

$$24^2 = 9^2 + x^2$$
$$x^2 = 24^2 - 9^2$$
$$= 576 - 81 = 495$$
$$x = \sqrt{495} = 22.248 + \text{ feet.} \quad \text{Ans.}$$

79. Relation of the Angles and Sides of a Triangle. In any triangle there are six parts—three sides and three angles. Three sides, two sides and one angle, or one side and two angles will determine the shape and size of the triangle. These relations are called *functions* of the angles, and the unknown parts of a triangle may be computed from the values of the known parts.

NOTE. This word *function* is used in mathematics to denote a quantity which depends upon some other quantity for its value. For example, the price of eggs depends mainly upon the supply and demand; hence the price of eggs is a function of the supply and demand. The area of a circle is a function of the radius; the distance a train can cover is a function of speed and time.

Let $M\,A\,N$, Fig. 29, be any acute angle. Then if lines be drawn from the points B, D, and F perpendicular to $A\,N$, the right triangles $A\,B\,C$, $A\,D\,E$, and $A\,F\,G$ will be formed. Such triangles, having their angles all equal, are called *similar*, and their corresponding sides are proportional.

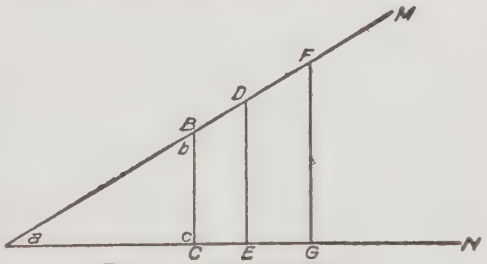


Fig. 29. Similar Triangles.

Therefore:

$$\frac{B\,C}{A\,B} = \frac{D\,E}{A\,D} = \frac{F\,G}{A\,F} \text{ and } \frac{B\,C}{A\,C} = \frac{D\,E}{A\,E} = \frac{F\,G}{A\,G}, \text{ etc.}$$

These ratios which depend upon the size of the angle a for their values are functions of a. There are six such ratios for every triangle and for convenience they have been given names. The most important of these are the following:

$\frac{BC}{AB}$ = sine of a (written sin a) $\frac{BC}{AC}$ = tangent of a (written tan a)

$\frac{AC}{AB}$ = cosine of a (written cos a) $\frac{AC}{BC}$ = cotangent of a (written cot a).

As the similar triangles ADE and AFG have also the angle a, the similar ratios of their sides may be expressed in the same way. Thus:

$$\frac{DE}{AD} = \sin a \qquad \frac{FG}{AF} = \sin a \qquad \frac{AE}{AD} = \cos a \qquad \frac{FG}{AG} = \tan a$$

In other words, sin a is always the same for the same angle, and if AM and AN were prolonged so that an infinite number of triangles might be formed, sin a would still retain the same value. The same plan may be followed with angle b, Fig. 29, with the result that $\sin b = \frac{AC}{AB}$, $\cos b = \frac{BC}{AB}$, and $\tan b = \frac{AC}{BC}$. If these values are compared with those given for a, it will be seen that $\sin a = \cos b$, $\cos a = \sin b$, $\tan a = \cot b$, etc. As the angles a and b are complementary, this may be considered the rule for complementary angles.

The rules for the functions of an angle may be stated thus:

$$\sin a = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \tan a = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\cos a = \frac{\text{adjacent side}}{\text{hypotenuse}} \qquad \cot a = \frac{\text{adjacent side}}{\text{opposite side}}$$

If the sides of the right triangle ABC, Fig. 30, have values of 3, 4, and 5, the values of the functions of a will be as follows:

$\sin a = \frac{3}{5} = .6$ $\tan a = \frac{3}{4} = .75$

$\cos a = \frac{4}{5} = .8$ $\cot a = \frac{4}{3} = 1.33+$

Fig. 30. Triangle with Lengths of Sides Given.

The table, page 170, will show that the angle whose sine is .60 is about 37°. The values of cos, tan, and cot also agree approximately with the table values for 37°. If 45° and 30°−60° triangles be taken, their functions are as easily obtained.

Examples. 1. Given $\sin a = \frac{5}{13}$, to find the value of the other side and the remaining functions.

SOLUTION.

The opposite side = 5, the hypotenuse = 13. Hence the other side is equal to $\sqrt{13^2 - 5^2} = 12$. The three remaining functions are $\cos a = \frac{12}{13}$, $\tan a = \frac{5}{12}$, $\cot a = \frac{12}{5}$. (Table gives $\cos \frac{12}{13} = 23° -$.)

2. Find the value of the sine, cosine, and tangent of 45°.

SOLUTION. A 45° triangle has its opposite and adjacent sides equal, hence if the hypotenuse is assumed to be 10, the sides may be calculated by the law of the right triangle, giving a value 7.07.

$$10^2 = x^2 + x^2 = 2x^2$$
$$x^2 = 50$$
$$x = 7.07+$$

Hence $\sin 45° = \frac{7.07}{10} = .707$

$\cos 45° = \frac{7.07}{10} = .707$

$\tan 45° = \frac{7.07}{7.07} = 1$

A careful trial of all possible angles between 0° and 90° would show that the functions may have the following values:

$\sin 0° = 0$, and increasing angles have increasing values up to $\sin 90° = 1$. (Verify sin, cos, and tan values from table, page 170.)

$\cos 0° = 1$, and increasing angles have decreasing values down to $\cos 90° = 0$.

$\tan 0° = 0$, and increasing angles have increasing values up to $\tan 90° = \infty$.*

80. Areas of Triangles. A triangle is a surface and as such has two dimensions, *base* and *altitude*. Under *denominate numbers* the fact was brought out that surfaces or areas were measured by square measure, and that the area of the surface was proportional to the product of its two dimensions. If the surface is a triangle it has been found that the *area is equal to one-half of the product of the base by the altitude*. For example, in Fig. 24 and Fig. 25, the area of each triangle is $\frac{1}{2}(B\,C \times A\,D)$. Other methods might be given, but by the above method, most cases which are met may be easily solved.

*Called the infinity sign, or "infinity," meaning infinitely large.

Examples. 1. Find the area of the triangle $A\,B\,C$ shown in Fig. 26 if the base $B\,C$ is 10 inches and the altitude $A\,B$ is 10 inches.

SOLUTION. Area $= \frac{1}{2}(A\,B \times B\,C) = \frac{1}{2}(10 \times 10) = 50$ square inches.

2. Find the area of a right triangle whose base is 50 feet and the hypotenuse 200 feet.

SOLUTION. To obtain the altitude, make use of the law of the right triangle, *i.e.*, take the square root of the difference in the squares of the hypotenuse and base of the triangle. $\overline{200}^2 - \overline{50}^2 = 40000 - 2500 = 37500$.

$\sqrt{37500} = 193.64+$ ft. altitude of the triangle.

Area of the triangle $= \frac{1}{2}(50 \times 193.64) = 4841$ sq. ft.

3. A ladder 40 feet long when placed on the ground 24 feet from the bottom of a wall reaches to the top. Determine the height of the wall.

SOLUTION. As the ladder is 40 feet long and the base is 24 feet from the wall, there is a right-angle triangle whose hypotenuse is 40 feet, one short side 24 feet, and the other short side to be determined. As in the previous example make use of the law of the right triangle and this will give $40^2 - 24^2 = 1024$. Taking the square root of 1024 will give 32 feet as the height of the wall.

PROBLEMS FOR PRACTICE

1. A triangular piece of sheet metal weighs 18 pounds. If the height of the triangle is 8 feet and the base 3 feet, what is the weight of the metal per square foot?

2. In surveying a triangular piece of land it was found that two of the angles measured 44° 56′ 4″ and 31° 11′ 8″. Compute the value of the third angle. Ans. 103° 52′ 48″

3. An iron brace used in supporting a shelf is fastened to the wall 18 inches below the shelf and to the shelf 12 inches from the wall. Find the length of the brace.

4. The area of a triangle is 24 square inches. If the altitude is 6 inches, find the length of the base.

5. An iron chimney is supported by a guy wire which makes an angle of 63° 24′ with the ground. Determine the angle between the chimney and guy wire. Ans. 26° 36′.

6. The grade of a railroad is such that in a horizontal distance of 5,000 feet the track rises 300 feet. What is the tangent of the angle between the track and the horizontal?

7. If one end of a ladder 50 feet long rests on the ground 10 feet from a wall, what is the cosine of the angle formed between the ladder and the ground? Find the sine and tangent of the same angle. Answers: .2; .98; 4.9.

8. Find the area of the right triangle of Problem 7.

QUADRILATERALS

81. A *quadrilateral* is a polygon bounded by four straight lines, as Fig. 31.

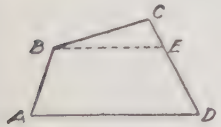


Fig. 31. Quadrilateral.

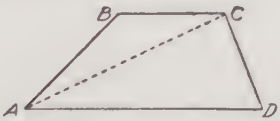


Fig. 32. Trapezoid.

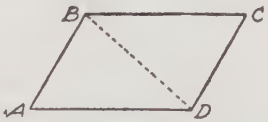


Fig. 33. Parallelogram.

A *trapezoid* is a quadrilateral having two sides parallel, Fig. 32. The parallel sides are called the *bases* and the perpendicular distance between the bases is called the *altitude*.

A *parallelogram* is a quadrilateral whose opposite sides are parallel, Fig. 33.

Fig. 34. Rectangle.

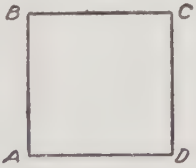


Fig. 35. Square.

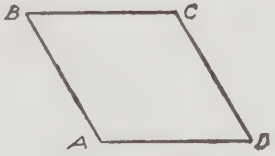


Fig. 36. Rhombus.

The three important kinds of parallelograms are as follows:

The *rectangle*, Fig. 34, whose angles are right angles.

The *square*, Fig. 35, all of whose sides are equal and whose angles are right angles.

The *rhombus*, Fig. 36, whose sides are equal but whose angles are not right angles.

82. **Areas of Quadrilaterals.** Considering first the *trapezoid*, it will be seen that in Fig. 32 the dotted line $A\,C$ (called a *diagonal*) divides the figure into two triangles, $A\,B\,C$ and $A\,C\,D$, whose

altitudes are the same, viz, the altitude of the trapezoid. By a previous rule the areas of these triangles will be respectively

$$\frac{1}{2}\ (\text{altitude} \times B\,C) \text{ and } \frac{1}{2}\ (\text{altitude} \times A\,D)$$

Therefore Area of trapezoid = altitude $\times \dfrac{B\,C + A\,D}{2}$.

Rule: (a) *The area of a trapezoid is equal to the product of the altitude by one-half the sum of the bases.*

The area of a *quadrilateral* like that shown in Fig. 31 may be obtained by drawing a line $B\,E$, parallel to the base $A\,D$. The figure will then be divided into a trapezoid and an extra triangle whose areas are measurable by the methods given.

The *parallelogram*, Fig. 33, may also be divided by means of a diagonal into two triangles whose areas are respectively $\dfrac{A\,D \times altitude}{2}$ and $\dfrac{B\,C \times altitude}{2}$. But $A\,D$ and $B\,C$ are equal and, consequently,

the sum of these areas $= \dfrac{2(A\,D \times altitude)}{2} = A\,D \times altitude.$

Rule: (b) *The area of a parallelogram is equal to the product of the base and altitude.* This will be true of any parallelogram and hence true of the rectangle, Fig. 34, the square, Fig. 35, and the rhombus, Fig. 36. In the case of the first two, the fact that the angles are all right angles makes the length of the vertical side become the altitude.

Rule: (c) *The area of a rectangle is equal to the product of two adjacent sides.* This has already been learned in denominate numbers. The fact that all land measurements are made in the rectangular system, makes the rectangle an extremely important figure.

Areas of Other Polygons. In finding the areas of polygons, other than the triangle and quadrilateral, the system of dividing the figure into triangles is used. For example, the regular *hexagon*, Fig. 37, consists of six equal equilateral triangles, and consequently the area of the hexagon will be $\dfrac{6\,(A\,B \times O\,L)}{2} = 3\,(A\,B \times O\,L)$.

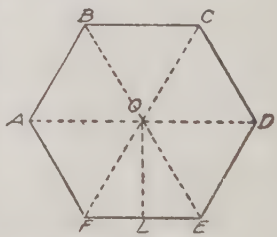


Fig. 37. Hexagon.

Rule: (d) *The area of a regular hexagon is equal to three times the product of one side (which is the same as $\frac{1}{2}$ the perimeter) by the perpendicular distance from the side to the center of the hexagon.*

PROBLEMS FOR PRACTICE

1. What is the area of a cross-section of a regular hexagonal drain tile whose sides are 2 inches in length and whose perpendicular distance from the center to a side is 1.7 inches?

2. How much would it cost to lay a concrete sidewalk 8 feet wide and 525 feet long at 20 cents per square foot?

3. How many square yards of plastering are needed for walls and ceiling of a room 10 × 12 feet and 9 feet high; the room contains a door $3\frac{1}{2} \times 7$ feet and two windows $3\frac{1}{2} \times 6$ feet? Ans. 50−sq. yd.

4. An irregular shaped room has two parallel sides, one 14 ft. 8 in. long, the other 20 ft. 3 in. long. One end of the room is perpendicular to the two sides and is 12 ft. 5 in. long. What will it cost to paint the floor at 25 cents per square yard? Ans. $6.02+

5. A trough two feet deep with equally sloping sides is constructed. One end is 5 feet across the top and 4 feet across the bottom. How many square feet of tin will be required for the end?

CIRCLES

83. A *circle* is a plane figure bounded by a curved line called the *circumference*, every point of which is equally distant from a point within called the *center*, Fig. 38.

A *diameter* of a circle is a straight line drawn through the center, terminating at both ends in the circumference.

A *radius* of a circle is a straight line joining the center with the circumference. All radii of the same circle are equal and their length is always one-half that of the diameter.

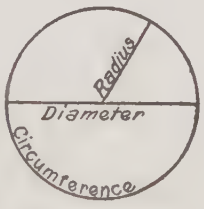


Fig. 38. Circle.

An *arc* is any part of the circumference of a circle.

An arc equal to one-half the circumference is called a *semi-circumference.*

A *chord* is a straight line joining the extremities of an arc, Fig. 39. When a number of chords form the sides of a polygon, the polygon is said to be *inscribed in the circle.*

A *segment* of a circle, Fig. 40, is the area included between a arc and a chord.

A *sector* is the area included between an arc and two radii draw to the extremities of the arc, Fig. 40.

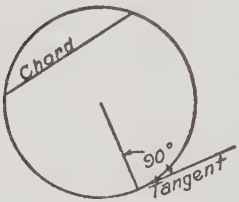


Fig. 39. Chord and Tangent.

A *tangent* is a straight line which touch the circumference at only one point, called th *point of tangency* or *point of contact*, Fig. 3 It may be proved that a tangent is perpendi ular to a radius drawn to the point of ta gency.

Concentric circles are circles having the sam center, Fig. 41.

Relation of the circumference to the diameter. If the lengths c the circumference and the diameter of a circle are carefully measure the ratio, $\frac{\text{circumference}}{\text{diameter}}$, will be found to have a fixed value, whatever circle may be selected. This constant ratio has been given the name π (Greek letter *pi*) and has a value of 3.1416 (approximately $\frac{22}{7}$). Therefore, *the circumference of any circle is equal to π times the diameter.* In most cases the value $\frac{22}{7}$ may be used with safety as the error is onl about .04%. Then, to find the circumference when the diameter known, *multiply the diameter by* $\frac{22}{7}$. Conversely to find the diameter when the circumferenc is known, *divide the circumference by* $\frac{22}{7}$.

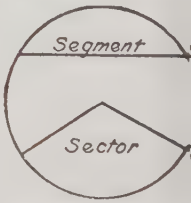


Fig. 40. Segment and Sector.

Fig. 41. Concentric Circles.

84. Areas of Circles. Let a regular hexago be inscribed in a circle, as shown in Fig. 4 It has already been found that the area of th hexagon is equal to one-half of OL times the sum of the 6 side *i. e.*, times the perimeter.

Now if the number of sides of the inscribed polygon shoul be increased, the perimeter would approach nearer to the circum

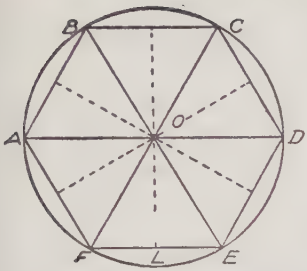


Fig. 42. Inscribed Hexagon.

ference and the line OL would approach the radius of the circumscribing circle. Therefore, if the number of sides should be increased indefinitely, the areas of the polygon and circle would be one and the same and equal to one-half the radius times the circumference.

$$\text{Area of circle} = \frac{\text{radius}}{2} \times \text{circumference.}$$

But the circumference of the circle $= \pi \times$ diameter $= \pi \times$ twice the radius. Hence

$$\text{Area of circle} = \left(\frac{\text{radius}}{2}\right) \times (2\pi \times \text{radius}) = \pi\ (\text{radius})^2.$$

or

$$\text{Area of circle} = \frac{\pi}{4} \times (\text{diam.})^2 = .7854\ (\text{diam.})^2$$

Rule: *The area of a circle is equal to the square of the radius multiplied by* $\frac{22}{7}$; *or is equal to the diameter squared times* .7854.

85. Areas of Sectors and Segments. The *area of a sector* is found by determining its fraction of the area of the corresponding circle. Thus, if the sector has an arc of 60°, the area of the sector will be $60 \div 360$ = one-sixth of the area of the circle, which has a radius equal to one of the sides of the sector. Similarly a sector with an arc of 45° has one-eighth the area of the corresponding circle.

There are many rules and tables for finding the *area of a segment.* The following formula is very simple and gives approximate results. The derivation of this formula is very difficult, as it involves higher mathematics; the formula may be used, however, by students and engineers who wish a simple expression giving approximate results.

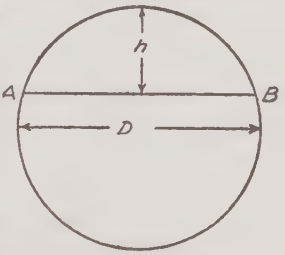


Fig. 43. Area of a Segment.

$$\text{Area } (A) \text{ of segment} = \frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$$

In this formula h = height of segment and D = diameter of circle, Fig. 43.

Example. 1. Find the area of a segment when the diameter of the circle is 12 feet and the height of the segment is 4 feet.

SOLUTION. In this case $h = 4$ feet and $D = 12$ feet. Substituting these values in the formula, the result obtained is 33 sq. ft.

$$A = \frac{4 \times 4^2}{3}\sqrt{\frac{12}{4} - .608}$$

$$= \frac{64}{3}\sqrt{3 - .608}$$

$$= \frac{64}{3}\sqrt{2.392} = \frac{64}{3} \times 1.55$$

$$= 33 + \text{square feet}$$

It has been shown that a line has but one dimension and a surface or area, two. If this be true, then any expression representing a length or area will always contain respectively one or two dimensions. For example, the expression for the length of the circumference of a circle is πD, which is the product of π, a pure number, and the diameter of the circle, a dimension. Again, the expression for the area of a circle is πR^2, in which the radius occurs *twice* giving the two dimensions necessary for an area; similarly, the expression for the area of a triangle ($\frac{1}{2}$ base $\times$ altitude) shows two dimensions. This fact is of use in proving up any expression. For instance, the expression given above for the area of a segment, could be judged as an area formula at once by noting that the dimension h occurs twice outside of the radical; and the dimension D in the numerator and h in the denominator of the fraction under the radical cancel each other.

PROBLEMS FOR PRACTICE

1. The diameter of a pulley is 18 inches. What is the circumference?

2. The bottom of a tank is circular and is 3 feet in diameter. What is the area of the bottom?

3. What is the area of one side of a flat circular ring whose outside diam. is 8 inches and inside diam. 4 inches? Ans. 38 – sq. in.

4. A carriage wheel 4 feet in diameter makes how many revolutions in traveling 1,000 feet?

5. The circumference of a chimney of circular cross section is 47.12 feet. What is its diameter? Ans. 14.9+ ft.

6. In patching the circular bottom of a steel tank 6 feet in diameter, a sector is cut out, the length of whose arc is $3\frac{1}{7}$ feet. Find the area of the patch required, disregarding the overlapping of the edges. Ans. 4.71 + sq. ft.

7. The bottom of a cylindrical cistern 9 feet in diameter is to be tinned. If sheets of tin 3 feet wide are available, find the areas of each of the 3 pieces used. The two end pieces have the shape of segments.

8. Find the number of square feet of glass required for a circular arched window whose height is 10 inches and the radius of whose arc is 24 inches. Ans. 1.89 + sq. ft.

SOLIDS

86. A *solid* has three dimensions—length, breadth, and thickness. The most common forms of solids are *prisms, cylinders, pyramids, cones,* and *spheres.*

PRISMS

87. A *prism* is a solid having two opposite faces, called *bases,* which are equal and parallel, and other faces, called *lateral faces,* which are parallelograms. The *altitude* of a prism is the perpendicular distance between the bases.

Prisms are called *triangular, rectangular, hexagonal,* etc., according to the shape of the bases. Further classifications are as follows:

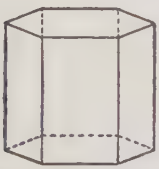


Fig. 44. Right Prism.

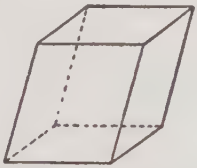


Fig. 45. Parallelopiped.

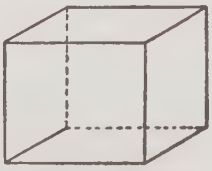


Fig. 46. Rectangular Parallelopiped.

A *right* prism is one whose lateral faces are perpendicular to the bases, Fig. 44.

A *regular* prism is a right prism having regular polygons for bases.

A *parallelopiped* is a prism whose bases are parallelograms, Fig. 45. If all the edges are perpendicular to the bases, it is called a *right parallelopiped.*

A *rectangular parallelopiped* is a right parallelopiped whose bases and lateral faces are rectangles, Fig. 46. When these faces are all squares the prism is called a *cube*.

88. **Areas of Prisms.** The *area of the bases* of any given prism is to be found by the method already given for that particular shape of base. The *lateral area*, as each face is a parallelogram, will be the sum of the products of each base line by the altitude. This is equivalent to the product of the perimeter of the base by the altitude. The *total area* will evidently be the sum of the areas of the two bases and the lateral faces.

Example. Find the total area of a box 3 feet long, 2 feet wide, and 6 feet deep.

SOLUTION.

The area of one base is 2×3 or 6 sq. ft., and of two, 12 sq. ft.

The perimeter of the base $= 2 + 2 + 3 + 3 = 10$ ft.; the altitude $= 6$ ft.

The lateral area $= 10 \times 6 = 60$ sq. ft.

The total area $= 60 + 12 = 72$ sq. ft. Ans.

89. **Volumes of Prisms.** It has already been made clear in the discussion of *Cubic Measure* in denominate numbers, that the measurement of volume involves the product of the three dimensions of the

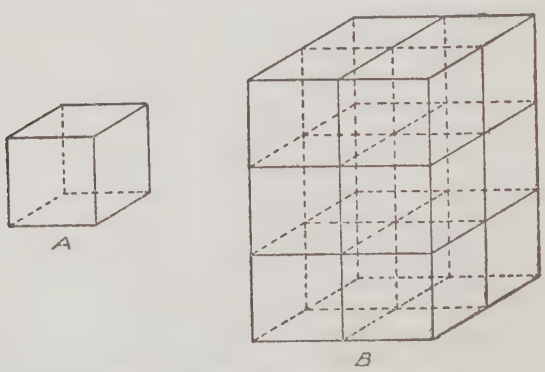


Fig. 47. Volume of a Rectangular Solid.

figure and is determined by the number of times the unit of cubic measure, such as the cubic inch, cubic centimeter, or cubic foot, is contained in the volume under consideration. If A, Fig. 47, represents a cubic foot, it will be noticed that A is contained four times

in the bottom layer of B. As there are three layers, A is contained a total of twelve times in B, *i.e.*, the volume of B is 12 cubic feet. But the area of the base of B is 4 sq. ft. and the altitude is 3 ft. The product of these two gives 12 cu. ft., the same as obtained by the use of the unit A. Rule: *The volume of any prism is equal to the product of the area of the base by the altitude.*

PROBLEMS FOR PRACTICE

1. What is the weight of a cast-iron block 6 inches long, 5 inches wide, and 3 inches high? The block is a rectangular parallelopiped and weighs .26 lbs. per cubic inch.

2. How many square feet of sheet copper will be required to make an open rectangular tank 7 feet long, 3 feet wide, and $1\frac{1}{2}$ feet deep, allowing 10% extra for waste?

3. How many cubic feet of concrete will be required to construct a column 15 feet high having a hexagonal cross-section. The cross-section is $1\frac{1}{2}$ feet long on each side of the hexagon and the perpendicular distance from center to each side is 1.3 feet. Ans. 87.75 cu. ft.

4. What will it cost to paint the above column, exclusive of the base and top, with waterproofing at 35 cents per square yard?

5. Find the volume of a triangular prism whose altitude is 29 feet, and each side of the base is 4 feet. Ans. 138.4 cu. ft.

6. Find the total area of the prism given in Problem 5.

CYLINDERS

90. A *cylinder* is a solid having as bases two equal parallel curved surfaces and as its lateral face the continuous surface generated by a straight line connecting the bases and moving along their circumferences. The bases are usually circles and such a cylinder is called a *circular cylinder*.

Fig. 48.
Right Cylinder.

A *right cylinder*, Fig. 48, is one whose side is perpendicular to the bases.

91. Areas of Cylinders. As a cylinder may be considered a prism with an infinite number of sides, *its lateral area is equal to the product of the circumference of the base by the altitude.*

The total area is evidently equal to the sum of the areas of the bases and the lateral area.

92. Volumes of Cylinders. *The volume of a cylinder is equal to the product of the area of one base by the altitude.*

Examples. 1. How much tin will be required to make a cylindrical can, 3 inches in diameter and 4 inches high, allowing nothing for seams and waste?

SOLUTION.

The can is a right cylinder the circumference of whose base is $\frac{22}{7} \times 3 = \frac{66}{7} = 9.43$ inches, nearly. Now, the radius of the base is $3 \div 2 = 1\frac{1}{2}$ inches, and the area of the base $\frac{22}{7} \times (1\frac{1}{2})^2 = 7.08$ sq. in., nearly.

The lateral area is $9.43 \times 4 = 37.72$ sq. in.

The total area equals $37.72 + (2 \times 7.08) = 37.72 + 14.16 = 51.88$ square inches.

2. A piece of iron weighing 30 pounds has 4 holes 6 inches deep and 1 inch in diameter bored into it. What is the weight of the piece after the boring has been completed, iron weighing .26 lbs. per cubic inch?

SOLUTION.

Each hole has a radius of $\frac{1}{2}$ inch and its volume is then $\frac{22}{7} \times (\frac{1}{2})^2 \times 6 = 4.7$ cu. in.

There are 4 holes making $4 \times 4.7 = 18.8$ cu. in.

The weight of the material cut out is $18.8 \times .26 = 4.9$ lbs.

Then the final weight is $30 - 4.9$ lbs. $= 25.1$ lbs.

PROBLEMS FOR PRACTICE

1. What is the capacity of a cylindrical tank 9 feet long and 5 feet in diameter, inside measurements?

2. If a gallon contains 231 cubic inches, what must be the diameter of a cylindrical tank 10 feet high which will hold 1,000 gallons? Ans. 4.1 + ft.

3. Find the number of square feet of material necessary for a straight piece of copper pipe 10 feet long and 12 inches in diameter.

PYRAMIDS

93. A *pyramid* is a solid whose base is a polygon and whose sides are triangles.

The vertices of the triangles meet to form the *vertex* of the pyramid.

The *altitude* of the pyramid is the perpendicular distance from the vertex to the base.

Pyramids are named according to the kind of polygon forming the base, viz, triangular, quadrangular, Fig. 49, pentagonal, Fig. 50, hexagonal, Fig. 51.

Fig. 49. Pyramid.

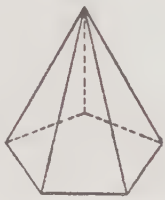


Fig. 50. Regular Pyramid.

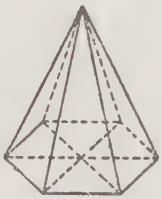


Fig. 51. Hexagonal Pyramid.

A *regular* pyramid is one whose base is a regular polygon and whose vertex lies in a perpendicular erected at the center of the base, Fig. 50.

The *slant height* of a regular pyramid is a line drawn from the vertex perpendicular to a side of the base. (See the line *O F*, Fig. 52.) In other words, it is the altitude of one of the triangles which form the sides.

The *lateral edges* of a pyramid are the intersections of the triangular sides, called the *faces*.

94. Areas of Pyramids. (a) The *lateral area* is the combined area of all the triangles forming the sides.

The area of each triangle is equal to the product of the base by one-half the altitude. Therefore the lateral area of a pyramid is found by adding the products of each side of the base by one-half the same altitude; *i.e.*, it is equal to the perimeter of the base multiplied by one-half the slant height. If the slant height is not given it can usually be found by means of the law of the right triangle.

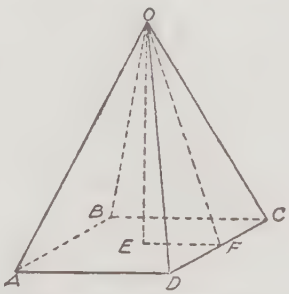


Fig. 52. Pyramid Showing Altitude and Slant Height.

(b) The *total area* of a pyramid is equal to the sum of the lateral area and the area of the base.

Example. Find the lateral area and total area of the pyramid shown in Fig. 52, if it has an altitude $O\,E$ of 12 feet and each side of the square base is 8 feet.

SOLUTION.

$E\,F = 4$ feet. Since the angle $O\,E\,F$ is a right angle

$$\overline{OF}^2 = \overline{OE}^2 + \overline{EF}^2$$
$$= 144 + 16$$
$$= 160$$
$$OF = \sqrt{160} = 12.65 \text{ (nearly)}$$
$$\text{The lateral area} = \frac{12.65 \times 32}{2}$$
$$= 202.4 \text{ square feet.} \quad \text{Ans.}$$

The area of the base is $8 \times 8 = 64$ sq. ft., making the total area $64 + 202.4 = 266.4$ sq. ft. Ans.

95. Volumes of Pyramids. It may be shown that any triangular prism may be divided into three equivalent triangular pyramids, having bases and altitudes equal to those of the prism; and therefore the volume of each pyramid must be one-third of the volume of the prism. But the volume of a prism is equal to the product of the area of the base by the altitude; therefore, *the volume of a pyramid is equal to the product of the area of the base by one-third of the altitude.*

PROBLEMS FOR PRACTICE

1. A triangular pyramid is 9 inches in height and each side of the base is 4 inches long. Find the volume.

2. What is the volume of a square pyramid one side of whose base is 4 inches and whose height is 4 inches?

3. Find the total area of a square pyramid whose slant height is 28 inches and one side of whose base is 8 inches.

4. A regular hexagonal pyramid has an altitude of 18 inches, and each side of the hexagon is 6 inches long. Find total area and volume of pyramid. Answers: 430.84 + sq. in.; 561.16 + cu. in.

CONES

96. A *cone* is a solid bounded by a conical surface and a plane which cuts the conical surface. It may be considered as a pyramid with an infinite number of sides.

The conical surface is called the *lateral area*, and the plane is called the *base*. The conical surface tapers to a point called the *vertex*.

The *altitude* of a cone is the perpendicular distance from the vertex to the base.

An *element of the cone* is any straight line from the vertex to the perimeter of the base.

Fig. 53. Circular Cone.

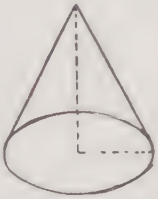


Fig. 54. Cone of Revolution.

A *circular cone* is a cone whose base is a circle, Fig. 53.

A *right circular cone*, or *cone of revolution*, Fig. 54, is a cone whose axis is perpendicular to the base. It may be generated by the revolution of a right triangle about one of the sides as an axis.

97. Areas of Cones. The *lateral area* of a cone is found in the same way as in the case of a pyramid. *Multiply the perimeter of the base by one-half the slant height.*

Example. The base of a cone is 12 inches in diameter and the slant height is 16 inches. What is the total area?

Solution.

$$\text{Perimeter of base} = 12 \times \frac{22}{7}$$

$$= 37.71 \text{ inches}$$

$$\text{Lateral area} = 37.71 \times \frac{16}{2}$$

$$= 301.68 \text{ square inches}$$

To find the *total area*, add to this the area of the base. In the above example, the area of the base is

$$\frac{22}{7} \times 36 = 113.14 \text{ sq. in.}$$

Total area = 113.14 + 301.68 = 414.82 square inches.

98. Volumes of Cones. As a cone may be considered a pyramid with an infinite number of sides, it follows from Section 95 that,

the volume of a cone is equal to the product of the area of the base by one-third the altitude.

Example. The altitude of a cone is 18 inches and the radius of the base is 2 inches. What is the volume?

$$\text{Area of base} = \frac{22}{7} \times 4 = 12.57$$

$$\text{Volume} = 12.57 \times 6 = 75.42 \text{ cubic inches}$$

In case the altitude is not known, it may be found if the slant height and the radius of the base are known.

PROBLEMS FOR PRACTICE

1. Find the volume of a cone whose altitude is 12 inches and the diameter of whose base is 6 inches.

2. Find the total area of a cone having an altitude of 21 feet and a base whose diameter is 15 feet. Ans. 702.4 sq. ft.

SPHERES

99. A *sphere* is a solid bounded by a curved surface, every point of which is equally distant from a point within called the *center*.

The *diameter* is a straight line drawn through the center and having its extremities in the curved surface. The *radius* is the straight line from the center to a point on the surface; it is equal to one-half the diameter.

A *plane is tangent to a sphere* when it touches the sphere in only one point. A plane perpendicular to a radius at its outer extremity is tangent to the sphere.

100. Areas of Spheres. To find the *area* of the surface of a sphere, *multiply the square of the diameter by* $\frac{22}{7}$.

Example. Find the area of the surface of a sphere which is 9 inches in diameter.

$$9^2 = 81. \quad 81 \times \frac{22}{7} = 254.6 \text{ square inches}$$

To find the *diameter* when the surface is given, *divide the surface by* $\frac{22}{7}$ *and extract the square root of the quotient.*

Example. What is the diameter of a sphere whose area is 113.14 square feet?

$$113.14 \div \frac{22}{7} = 36 \text{ nearly.} \quad \sqrt{36} = 6 \text{ feet}$$

101. Volumes of Spheres. To find the *volume* of a sphere, multiply the cube of the diameter by $\frac{\pi}{6}$ (equal to $\frac{11}{21}$ nearly).

Examples. 1. A sphere is 10 inches in diameter; what is the volume?

$$10^3 = 1000. \quad 1000 \times \frac{11}{21} = 523.8 \text{ cubic inches}$$

2. Find the volume of a sphere having a diameter of 7 feet.

$$7^3 = 343; \quad 343 \times \frac{11}{21} = 179\tfrac{2}{3} \text{ cubic feet}$$

3. A certain sphere contains 8,184 cubic feet. Find the diameter.

SOLUTION.

To find the diameter when the volume is known, divide the volume by $\frac{11}{21}$ and find the cube root of the quotient (by Logarithms).*

$$8184 \div \frac{11}{21} = \overset{744}{\cancel{8184}} \times \frac{21}{\cancel{11}_{1}} = 15624$$

$$\sqrt[3]{15624} = 25 \text{ ft. nearly.} \quad \text{Ans.}$$

An examination of the different expressions for *volume* given in the preceding sections, in a manner similar to that given on Page 118, will show that each expression contains three dimensions.

PROBLEMS FOR PRACTICE

1. The surface of a sphere contains 314.16 square inches. What is the diameter?

2. What will be the volume of an aluminum sphere which has a diameter of 15 inches? Ans. 1.02 cu. ft.

3. How much will a sphere of cast iron weigh if it is 3 feet in diameter and iron weighs 450 lbs. per cu. ft.?

4. How much will it cost to paint the sphere in Problem 3 at 25 cents per square yard?

5. How much less surface has a sphere which is 12 inches in diameter than a cube with an edge of 2 feet? Ans. 20.85 + sq. ft.

6. A spherical balloon, when inflated, has a diameter of 50 feet. Find the surface and the number of cubic feet of gas contained in the bag.

*Reserve this solution until the next section has been studied.

LOGARITHMS

102. Methods of Calculation by Logarithms. The use of logarithms greatly simplifies multiplication, division, raising quantities to higher powers, and finding roots of quantities. It is not, however, our purpose at this time to try to explain how logarithms are derived, or to make clear why *adding logarithms multiplies the numbers and subtracting logarithms divides them.* Leaving this explanation to a later part of the course, it is sufficient at this point to explain carefully the method of calculation, and to give many examples of the manner of using the logarithmic tables on pages 171 and 172.

103. Characteristic and Mantissa. The logarithm of a number consists of two parts. One part at the left of the decimal point is called the *characteristic* and has a positive, zero, or negative value, depending upon the size of the number. The other part at the right of the decimal point is called the *mantissa,* and is a number depending wholly on the figures which form the number whose logarithm is desired. The characteristic of any number between 1 and 10 is 0; that is, such numbers as 2, 4.62, 7.845, 9.57 have a zero characteristic. Numbers between 10 and 100, such as 10.2, 33.42, 85.236, have a characteristic of 1. Numbers between .1 and 1 have a characteristic of -1, etc. Putting this into the form of a rule: *Any number equal to or greater than* 1 *has a characteristic which is one less than the number of digits in the number; any number less than* 1 *has a negative characteristic which is one more than the number of zeros between the decimal point and the first numerical figure.*
For example

36.4	has a characteristic	1
523	" " "	2
532,654	" " "	5
1.532	" " "	0
.0072	" " "	-3
.000038	" " "	-5
10	" " "	1

Practice this rule by finding the characteristics of a great many numbers until the process is clear to you.

The mantissa is the part of the logarithm to the right of the decimal point and has a certain value for every succession of figures. For example, the mantissa is the same for 2734, 27.34, 2,734,000 and .002734. The value of the mantissa must always be obtained from the logarithm tables and may be found as follows:

Referring to the portion of the logarithm table given below it will be noticed that the column of figures at the extreme left consists of two figures each from 10 to 19.

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3 4 5	6 7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 8 12 17 21	25 29 33 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 11 15 19	23 26 30 34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 10 14 17	21 24 28 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 6 10 13 16	19 23 26 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9 12 15	18 21 24 27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 8 11 14	17 20 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8 11 13	16 18 21 24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2 5 7 10 12	15 17 20 22
18	2553	2577	2601	2625	2648	2672	2696	2718	2742	2765	2 5 7 9 12	14 16 19 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7 9 11	13 16 18 20

Next there are ten columns of four figures each, headed by the digits from 0 to 9 inclusive. These four-figured numbers are the mantissas of the numbers made up of the two figures at the left and a third at the top of each column. For example, the mantissa of 118 is found by taking the number 11 in the left-hand column and moving horizontally to the column headed 8 which is the third figure of the number 118; the mantissa sought is .0719. Remember that .0719 is also the mantissa of 1.18, 11.8, .00118, or 11,800. Again for 156,000, we move horizontally toward the right from 15 to the vertical column 6 and find the mantissa to be .1931.

By consulting the complete tables, pages 171,172, it will be seen that this system is easily extended to include the mantissa for any succession of *three numbers* whatsoever. A *fourth* figure may be found by making use of the nine columns at the extreme right as follows: Suppose we wish to find the mantissa of 1648. The mantissa for 164 is found by the method already explained and proves to be .2148. For the fourth place (viz, 8 in the number 1648) move horizontally from .2148 as far as column 8 at the extreme right and find 21. We must therefore add 21 to .2148 making the final mantissa .2169. The mantissa for 1893 is .2765 + 7 = .2772.

This process enables us to find any succession of four figures and for that reason these logarithm tables are called "four-place" tables. Tables of six and eight places are more complicated and are only used for the more accurate work. It should be mentioned here that if the student checks the results obtained by the four-place tables with the actual multiplication or division of the numbers, he will find slight differences between these results and those obtained from the logarithm tables. These differences, however, are comparatively small, and may be neglected as they are well within the per cent of error permitted in practical work.

104. Finding the Logarithm of a Number. It only remains to put the characteristic and mantissa together in order to complete the logarithm. For example, referring to the numbers indicated on the portion of the logarithmic table already given, the number 118 has a characteristic of 2 and a mantissa of .0719. The logarithm (or log "for short") of 118 is, therefore, 2.0719. The number 156,000 has a characteristic of 5 (one less than the number of places) and its mantissa is .1931. The complete log for 156,000 is 5.1931. Without further analysis it may be seen that the log of 1648 is 3.2169 and the log of 1893 is 3.2772.

Following this process, verify the following logarithms from the table.

Number	Logarithm	Number	Logarithm
3.46	0.5391	473,600	5.6754
.00856	−3.9325	872.6	2.9408
.9642	−1.9842	.0998	−2.9991
24.63	1.3914	6.843	0.8353
10,800	4.0334	34,970	4.5437

It often occurs in the use of four-place tables that the logarithms of numbers having five or more figures are desired. In this case increase the fourth place of the number by 1 if the fifth place is greater than 5, and drop the fifth and following places if the fifth place is less than 5. Thus the number 36.547 becomes 36.55 and its logarithm may be found from a four-place table. Similarly 2.7384 becomes 2.738.

105. Finding the Number From its Logarithm. The reverse process of finding the number which corresponds to a given logarithm is easily accomplished. Let us take the logarithm 2.7679 and find the corresponding number. Consulting the portion of the logarithm table shown below, we find that the mantissa .7679 is the fourth mantissa in the column headed 6. Therefore, 6 is the third figure of the number sought and moving horizontally to the last column at the left we find 58. This means that the mantissa .7679 corresponds

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	6	7	
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	6	7	
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	6	7	
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	7
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6

to the succession of figures 586. As the characteristic is 2 and a number with such a characteristic must consist of three numerical places, 586 is the number. Again, take the log 1.7846. The mantissa .7846 is in the column headed 9, and is the mantissa for the succession of figures 609. As the characteristic is 1 the number is 60.9.

When the mantissa of the logarithm given falls between two consecutive mantissas of the table, the columns at the right must be used. Let us find the number corresponding to the log -2.7908. Searching the columns no mantissa of this exact value is found; the next smaller mantissa is 7903, the difference being 5. Moving horizontally to the right from 7903, the difference 5 is found in the column headed 7. Therefore, 7 is the fourth place in the number sought and the succession of figures for the mantissa .7908 is 6177. As the characteristic is -2, the number sought must have one cipher between the decimal point and the first figure; the number is, therefore, .06177.

Following this process verify the correctness of the numbers for the following logarithms:

Given Logarithm	Mantissa in Table Next Smaller than given Mantissa	Difference	Figures Corresponding to given Log	Number Corresponding to given Log
4.5755	.5752	3	3763	37630
−1.7860	.7860	0	611	.611
3.0000	.0000	0	100	1000
0.9188	.9186	2	8293*	8.293
−3.6289	.6284	5	4255	.004255
0.4442	.4440	2	2781	2.781

*When two of the right-hand columns contain the same number, choose the smaller.

106. Multiplication by Logarithms. Now, suppose we wish to find by logarithms the product of two numbers like 251 and 3.16. The mantissa of 251 is .3997; the characteristic is 2; and the complete log. 2.3997. Similarly the log. of 3.16 is found to be 0.4997. Adding these two logarithms we have 2.8994, which is found from the table to be the log. of 793.2.

Logarithms should not always be used as they are sometimes not as simple as plain multiplication; for instance, $1{,}000 \times 3.16 = 3{,}160$, which result can be obtained mentally, but the product of two such numbers as 7,853 and 522 could not be so easily calculated. The characteristics of these numbers are respectively 3 and 2 (in all cases, *one less* than the number of places in the whole number) and by consulting the table the mantissas are respectively .8951 and .7177. The product of 7,853 and 522 is found, therefore, by first adding 3.8951 and 2.7177, giving 6.6128, then finding the number (410) whose mantissa is .6128 and finally pointing off 7 places (one more than the characteristic) in the result, giving 4,100,000 as the product.

Examples. 1. Find the product of 24 and 7,635.

Solution. The characteristics of the numbers are 1 and 3. The mantissas are found to be .3802 and .8828. *Adding the logarithms* gives 5.2630, which is found to be the logarithm of 183,200, the required product.

Log. 24 = 1.3802
Log. 7635 = 3.8828
Log. 183,200 = 5.2630

2. Find the product of 8,995 and 6,987.

SOLUTION. As both numbers are greater than 1,000 and less than 10,000, their characteristics are 3. The table shows for 8,995 a mantissa = .9540, and for 6,987, a mantissa = .8443. *Adding the "logs"* gives 7.7983. The table gives 6,284 as the number whose mantissa is .7983, and a characteristic of 7 makes the number 62,840,000.

Log. 8995 =	3.9540
Log. 6987 =	3.8443
Log.62,840,000 =	7.7983

3. Find the product of 42.873 and .542.

SOLUTION. As the tables give only four places, the last figure of the first number is neglected. If it had been 5 or more, the fourth place would have been increased by 1. It must be remembered that the characteristic depends only on the *whole number*, while the mantissa disregards the decimal point entirely.

Log. 42.873 =	1.6321
Log. .542 =	− 1.7340
Log. 23.23 =	1.3661

107. Division by Logarithms. As division has already been defined as the reverse of the process of multiplication, division by means of logarithms is accomplished by *subtracting* the log of the divisor from the log of the dividend. This operation presents no difficulties except in case the characteristic of the subtrahend exceeds that of the minuend or in case a negative characteristic occurs in the final result. See Problem 2.

Examples. 1. Divide 8,754 by 37.6.

SOLUTION. The logarithms are found in the same manner as in the previous examples, but the logs. are subtracted in this case, giving 2.3670, which is the log. of 232.8, the correct quotient.

Log. 8754 =	3.9422
Log. 37.6 =	1.5752
Log. 232.8 =	2.3670

2. Divide 2.34 by 865.

SOLUTION. In this example the dividend is smaller than the divisor and hence the subtraction results in a *minus* characteristic. This gives two ciphers to the left of the first numerical place, making the result of the division .002705. The minus characteristic is sometimes avoided by adding 10 to the characteristic of the dividend and *indicating* a subtraction of 10 after the mantissa; thus, 0.3692 = 10.3692 − 10 and the subtraction now gives 7.4322 − 10 which is equal to − 3.4322. The second method often avoids confusion and is usually to be preferred.

Log. 2.34 =	0.3692
Log. 865 =	2.9370
Log. .002705 =	− 3.4322

108. **Powers.** The power of a number is found by multiplying the logarithm of the number by the exponent. The number corresponding to this new log. is the desired power.

Examples. 1. Find the fourth power of 24.

Log. 24 = 1.3802
4
5.5208
Number = 331800

2. Find the fifth power of 23.

Log. 23 = 1.3617
5
6.8085
Number = 6434000

3. Find the seventh power of 0.8762.

Log. 0.8762 = 9.9426 − 10
7
69.5982 − 70 = $\bar{1}$.5982
Number = .3965

109. **Roots.** The root of a number is found by dividing the logarithm of the number by the index of the root. The number corresponding to this new log. is the desired root.

Examples. 1. Find the cube root of 8,753.

SOLUTION. The log. of 8,753 is 3.9421 and dividing this by 3 gives 1.3140, which is the log of 20.6, the desired cube root.

Log. 8753 = 3.9421
3)3.9421
Log. 20.6 = 1.31403

2. Find the square root of 59.5.

SOLUTION. The division by 2 is carried out as if no decimal point was in the log., giving .8873, which is the log. of 7.713, the required root.

Log. 59.5 = 1.7745
2)1.7745
Log. 7.713 = .8873

3. Find the sixth root of .9873.

SOLUTION. To avoid the minus characteristic, add and subtract some multiple of 10 which will contain 6 a whole number of times. Therefore, adding and subtracting 30 and dividing by 6 gives − 1.9990, which is the logarithm of .996.

Log. .9873 = − 1.9944
− 1.9944 = 29.9944 − 30
6)29.994 − 30
4.9990 − 5
Log. .996 = − 1.9990

4. Solve $\sqrt{6539}$

Log. 6539 = 3.8155 ÷ 2 = 1.9077
Number = 80.85

5. Solve $\sqrt[7]{3.65}$

Log. $3.65 = 0.5623 \div 7 = 0.0803$
Number $= 1.203$

6. Solve $\sqrt[4]{(367)^5}$

$(367)^5 = 5 \times \text{Log. } 367 = 5 \times 2.5647 = 12.8235$

Here we must take the fourth root of $(367)^5$ and hence must divide its logarithm by 4.

$12.8235 \div 4 = 3.2059$. Number $= 1607$

7. Solve $\sqrt[6]{\frac{(1278)^8}{7^4}}$

(First calculate the quantity under the root sign.)

$8 \times \text{Log. } 1278 = 3.1066 \times 8 = 24.8528$
$4 \times \text{Log. } \quad 7 = 0.8451 \times 4 = \ 3.3804$
21.4724

In order to extract the 6th root it is necessary to divide by 6, obtaining 3.5787, for which the number is 3791.

8. Solve $4722 \times \sqrt[3]{68}$

Log. $4722 = 3.6741$
$+\frac{1}{3}$ Log. $\quad 68 = 0.6108$
4.2849
Number $= 19270$

Special care must be taken in problems like the following:

$$(231 \times 244) - (113 \times \sqrt[3]{196})$$

Subtraction can not be simplified by logarithms, therefore, only two quantities, the minuend and subtrahend, can be calculated separately by means of logarithms, finding their respective numbers, and subtracting one from the other. Thus:

Log. $231 = 2.3636$
Log. $244 = 2.3874$
4.7510 Number $= 56360$

Log. $113 = 2.0531$
$+\frac{1}{3}$ Log. $196 = 0.7641$
2.8172 Number $= 656.4$
Difference 55703.6

Rules. (a) *The logarithm of a power of* 10, *positive or negative, is a whole number, the mantissa being zero in every case.*

(b) *If the logarithm is negative—true for every number between* 0 *and* 1*—it is customary to add* 10 *or some multiple of* 10 *to the characteristic and to indicate a subtraction of the same number from the mantissa.* Thus, -3.7583 becomes $7.7583 - 10$; -13.8625 becomes $7.8625-20$.

(c) *If a number is greater than* 1, *the characteristic of its logarithm is one unit less than the number of figures to the left of the decimal point.*

(d) *If the number is less than* 1, *the characteristic of its logarithm is negative and one unit more than the number of zeros between the decimal point and the first significant figure of the given number.*

(e) *Conversely, if the characteristic of a given logarithm is positive, make the number of figures in the integral part of the corresponding number one more than the number of units in the characteristic.*

(f) *If the characteristic is negative, make the number of zeros between the decimal point and the first significant figure of the corresponding number one less than the number of units in the characteristic.*

(g) *The logarithms of all numbers which differ from each other by powers of* 10 *have the same mantissa.*

PROBLEMS FOR PRACTICE

Find the logarithms of the following:

1. 53
2. 1,068
3. 821
4. 425
5. 650
6. 897 Ans. 2.9528

Find the numbers corresponding to the following logarithms:

7. $8.7284-10$ Ans. .0535
8. 1.8136
9. 6.9956
10. $7.5862-10$ Ans. .003856
11. 0.8744
12. 0.0085 Ans. 1.02

Solve the following problems by logarithms:

13. $\sqrt{826}$ Ans. 28.74
14. $\sqrt[3]{846}$
15. $\sqrt[5]{(381)^3}$ Ans. 35.36
16. $\sqrt[4]{\frac{(1021)^5}{(6)^3}}$ Ans. 1505
17. $5{,}026\times\sqrt[3]{40}$
18. $(1{,}030)^4\times\left(\frac{99}{108}\right)$

CURVE PLOTTING

110. Scope and Use of Curves. Graphical representation is the "shorthand" of the engineer, architect, and machinist. Everyone is more or less familiar with the detail drawings which the architect and the contractor find so useful in showing the exact location of the different parts of a building. The machinist, too, easily works up a complicated piece of mechanism with only plans, elevations, and sections to follow. There is another graphical method of showing the true relation of quantities, which is not so well known but which is none the less important, namely, *curves*. Like mechanical drawings, curves must be studied and the principles used in their construction must be learned before the proper meaning can be obtained from them, but with a little practice they become extremely useful not only in showing how one quantity changes with another, but in furnishing at a moment's notice information which would require hours of calculation to produce.

The range of application is unlimited. For example, curves can be plotted showing the cost of doing contract work or of operating a certain machine; the efficiency of a gang of men; the variations of temperature in a fever patient in a hospital ward; or the cost of high living. So accurate can these curves be drawn that the designer can use them in determining the allowable stress in structural steel girders, or the contractor at a glance can tell the cost per foot for any diameter of sewer pipe placed at any depth. Thus the student can readily see that curves are valuable records for commercial and professional men, for trained engineers and skilled machinists. The use of curves and their meaning and the methods of constructing them are clearly shown in the following illustrations.

PLOTTING METHODS

111. Co-Ordinate Paper. The curve in Fig. 55 has been drawn upon paper provided with parallel ruled lines; the lines drawn in both the horizontal and the vertical directions are equal distances apart, thus dividing the paper into small squares. The paper is called cross-section, or co-ordinate, paper and can either be ruled by the student or purchased already ruled at any stationer's store.*

*A limited supply of co-ordinate paper will be supplied by the School when needed, and students are strongly recommended to use it in practice work. Additional sheets will be sent on application at 10 cents per dozen sheets.

For convenience in plotting curves the side of each small square is one-tenth of an inch.

112. **Relation of Co-Ordinates to Curve.** After selecting co-ordinate paper the first step in plotting curves consists in drawing a vertical line near the left edge of the paper and a horizontal line near the bottom. These two lines, shown as A and B in Fig. 55, are called *axes* and intersect in a point called the *origin*—sometimes called the *zero point*. The vertical line is called the *vertical axis* and the horizontal line the *horizontal axis*.

Referring to Fig. 55, it can be seen that the series of points P_1, P_2, P_3 — P_8 lie upon the curve, and, further, the positions of these points, since they lie upon the curve, determine the shape of the curve. If this curve remains in the same relative position as regards the horizontal and the vertical axes, each point of this curve must have a definite position in relation to these axes. For example, the point P_2, to fall upon the curve, must be located 32 spaces up from the horizontal axis, and 10 spaces out to the right of the vertical axis. Again, point P_7 must be measured 20 spaces up from the horizontal axis and 35 spaces to the right of the vertical axis.

If we supplied other values than these given for P_2 and P_7, the student can readily see that the points P_2 and P_7 would not fall upon the curve but at some point either to the right or to the left of it. If this is so for two points, it will be so for any point on this curve or on any other curve; therefore, for any point on any curve, two values, one horizontal and one vertical, are necessary to locate a point, and these values, called the *co-ordinates of the point*, can only be used for locating this particular point. The distance measured on the vertical axis is called the *vertical value* and the distance on the horizontal axis the *horizontal value*.

The term "plotting the curve" is, therefore, understood to mean the locating of the points of any curve by the use of co-ordinates of these points.

Co-Ordinate Values. The small spaces, as measured on the vertical and on the horizontal axis, are usually drawn to a certain scale but may represent any values whatsoever. For example, one space on either axis may represent one foot, one yard, one year, one dollar, or any value that we choose to give it. If days be plotted or laid off on the horizontal axis and one day be represented by one

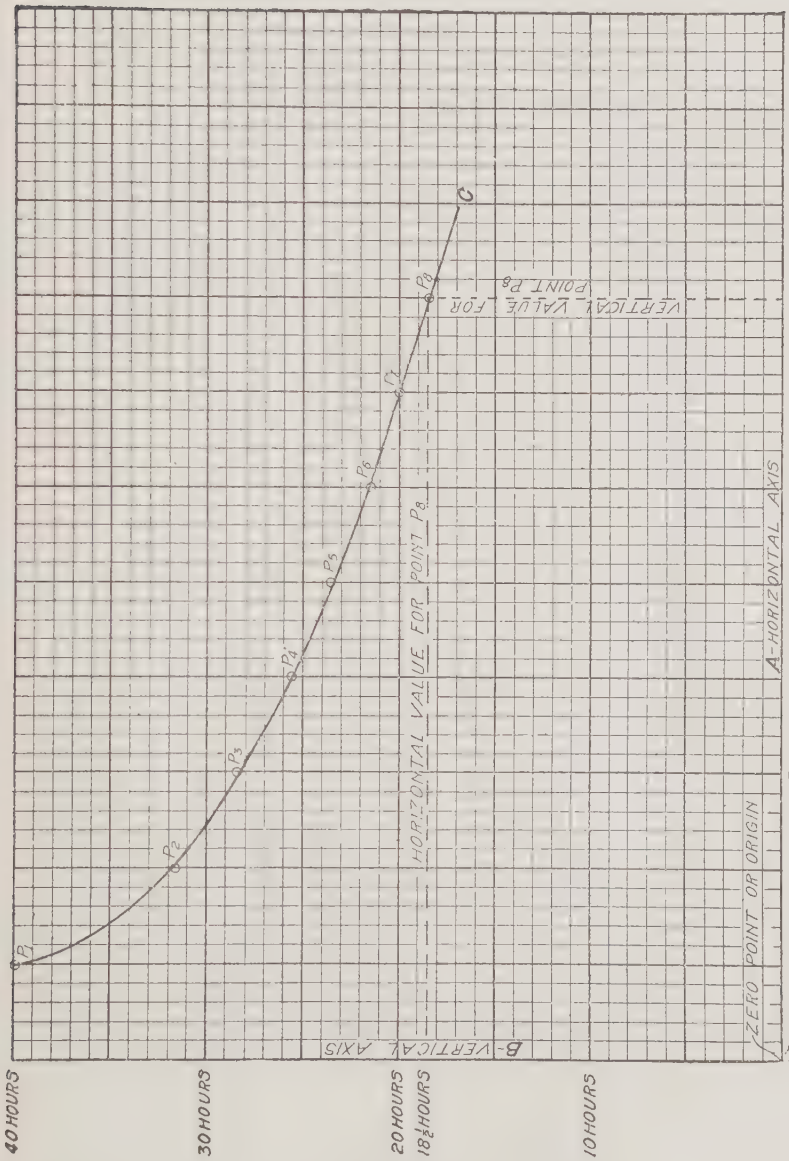


Fig. 55. Textbook Study Record Curve

space to the right of the origin, ten days are truly represented by ten of these spaces to the right of the origin, and so on for any number of days. Again, it may be stated that one space equals ten years, in which case ten spaces will represent 10×10, or 100 years.

113. Plotting the Curve. *Study Record Curve.* It was shown that each point on the curve must have for its location two values, viz. one on the horizontal and the other on the vertical axis. Suppose, for example, a student wished to keep a record of his study hours spent on the first eight textbooks of his course. The two quantities to be studied by this curve are *hours* and *textbooks*. After looking over his study record card, he finds that he completed the first textbook in 40 study hours, the second in 32 hours, the third in 28.5 hours, the fourth in 25.5 hours, and so on as given above.

Textbooks	Study Hours
1	40
2	32
3	28.5
4	25.5
5	23.5
6	21.5
7	20
8	18.5

To plot this curve, lay off the hour values on the vertical axis, Fig. 55, letting each space represent one hour; then for 40 hours count 40 spaces from the origin on the vertical axis. It is a good plan to write 10 hours, 20 hours, etc., just to the left of the vertical axis, as shown in the figure, to indicate how far up to go in order to get a certain number of hours without actually counting each space. Similarly, on the horizontal axis lay off the number of textbooks to a scale of one textbook for every five spaces, writing the number just under the horizontal axis, as in Fig. 55. If the study card, or the table shows that it requires 40 hours of study on textbook No. 1, these values represent the co-ordinates of P_1. Plot this point by reading from the origin to the right on the horizontal axis as far as textbook No. 1, then reading up the vertical line passing through this point as far as the horizontal line representing 40 hours; the intersection of the vertical and the horizontal lines is P_1, the point on the curve. Make a firm dot, mark it P_1, and go to the next set of values. The table shows that the second textbook was completed in 32 hours. Read out 10 spaces to the right of the origin on the horizontal axis and up the vertical line to the horizontal line representing 32 hours; place a dot at the intersection of these two lines, thus locating point P_2. Continue in exactly the same manner until all points have been plotted for the eight

textbooks. After all points have been plotted, draw a smooth line through the points and obtain the curve C. If the student is careful, he can draw in this curve freehand with a lead pencil, but in order to make a smooth line, an irregular curve of the type shown in Fig. 56 may be used to advantage. These curves can be purchased at any stationery store for a small sum. It is well to have one curve as well as a good ruler for this work.

Transmission Rope Curve. To further illustrate the plotting materials let us take another practical case. The engineering

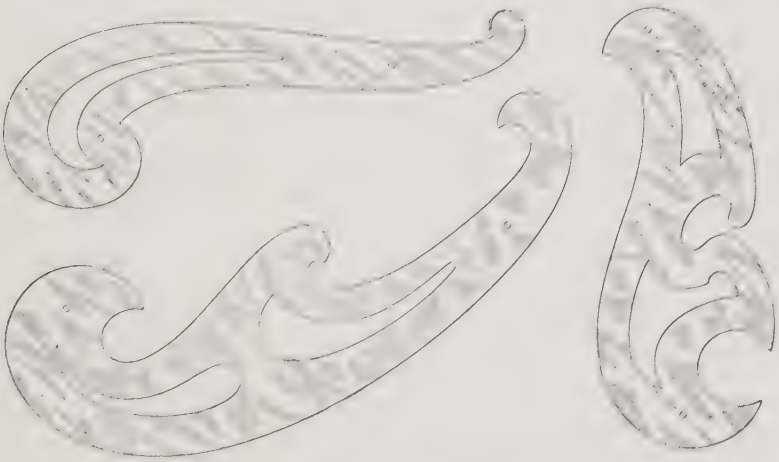


Fig. 56. Typical Irregular Curves. These are made of Wood, Hard Rubber, or Celluloid

department of a certain firm manufacturing transmission steel rope, having made tests of the horsepower developed by rope running at certain velocities, expressed this relation in the form of a curve. The velocities of the rope and the corresponding horsepowers developed are arranged in tabular form above the curve, Fig. 57.

The velocity of the rope in feet per second is chosen as the vertical value and the corresponding horsepower as the horizontal value, using a scale of 1 space on the vertical axis to equal 2 feet per second, and 1 space on the horizontal axis to equal 4 horsepower. To plot point P_1, lay off to scale on the horizontal axis 27 horsepower, remembering that one square equals 4 horsepower; that is, 27 horsepower will equal $\frac{27}{4}$, or 6.75, spaces. For the vertical value, since each

space equals 2 feet per second, 10 feet per second equals $\frac{10}{2}$, or 5, spaces. Therefore, to plot point P_1, read to the right on the horizontal axis $6\frac{3}{4}$ spaces and from this point up in a vertical line five spaces, placing a dot at this point to mark P_1 on the curve. Again, to plot point P_2, read to right from origin $\frac{53}{4}$, or 13.2, spaces and up on the vertical line at this point $\frac{20}{2}$, or 10, spaces. Continue in the same manner until all the points are plotted. For the convenience

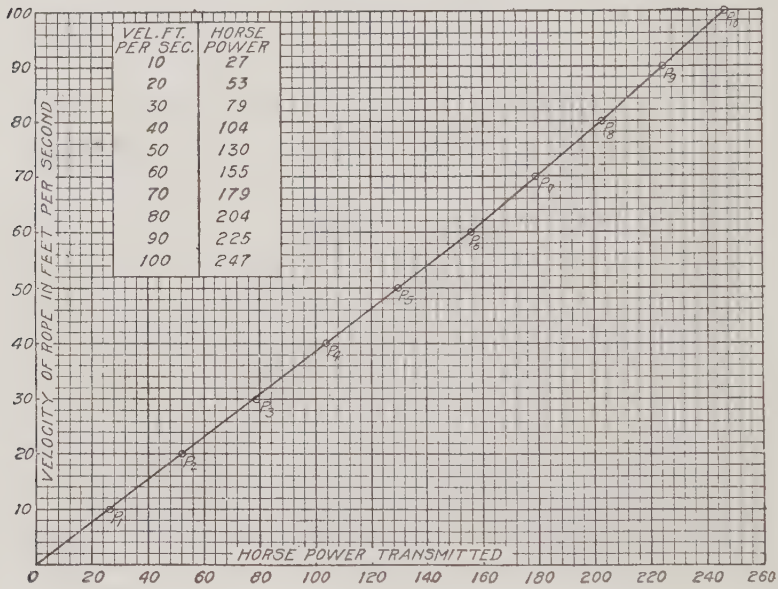


Fig. 57. Horsepower Curve for Transmission Steel Rope

of plotting and reading values, it is sometimes a good policy, before plotting any of the points, to lay off on the vertical axis all of the vertical values; that is, in this case 10 feet per second, 20 feet per second, 30 feet per second, etc. Then lay off to scale on the horizontal axis the horsepower values, as 27, 53, 79, etc., until all values up to 247 have been located. After the vertical and the horizontal values have all been marked, it is a very simple matter to establish their intersections and thus locate the points. This curve will be found practically a straight line up to point P_5, from which point the curve, although still a straight line, has a slightly steeper slope. Such a curve can be drawn with a straight edge or rule.

The student will note in this case and elsewhere that the term *curve* refers to the line drawn through the points whether such line proves to be a straight line or series of straight lines, or a curved line. Do not be confused, therefore, at the shape of your curve, and think because it is a straight line you have the wrong result. In most problems you will find, instead of regular curves or straight lines, the curve is made up of a series of straight lines between the points as shown in the Sunday load diagram, Fig. 65.

MEANING OF CURVES

114. Study Curve. The student will at first see no use for curves and, therefore, he should make a careful study of the curve, Fig. 55. First note that the curve drops gradually from P_1 to P_8. The question arises, "What does this mean?" This question can best be answered by asking another one. What sort of curve would have been obtained if the student had taken 40 hours for each textbook? Evidently such a curve would have been a straight line running along the 40 mark parallel with the horizontal axis. The drop in the curve towards the horizontal axis indicates that the student is taking less time to complete each succeeding text, the figures showing 40 hours for the first and 18.5 hours for the last. In other words, this curve is a graphical picture of the student's efficiency since he began to study. The completion of each text has contributed its share to his brain development and added to his knowledge of the subject, making it easier for him to grasp the subject matter of his course with each completed text. Further, it shows at a glance how many hours he required to finish any one text.

115. Horsepower Curve. The curve, Fig. 57, shows that, as the velocity of the rope in feet per second increases, the horsepower increases in the same proportion up to a point between points 5 and 6. From this point to point 9 the curve rises at a greater angle, which shows that less horsepower is produced for the same increase in velocity of rope. The fact that the curve continues as a straight line shows that the horsepower is still proportional to the velocity of the rope, but that this proportion is slightly different from that of the first part of the curve.

The student has now seen that each curve is a graphical pic-

ture of the relation of certain values which make up the curve. In other words, it is easier to use the curve than to wade through the mass of data making up the curve in order to obtain certain information regarding the action of one value on another.

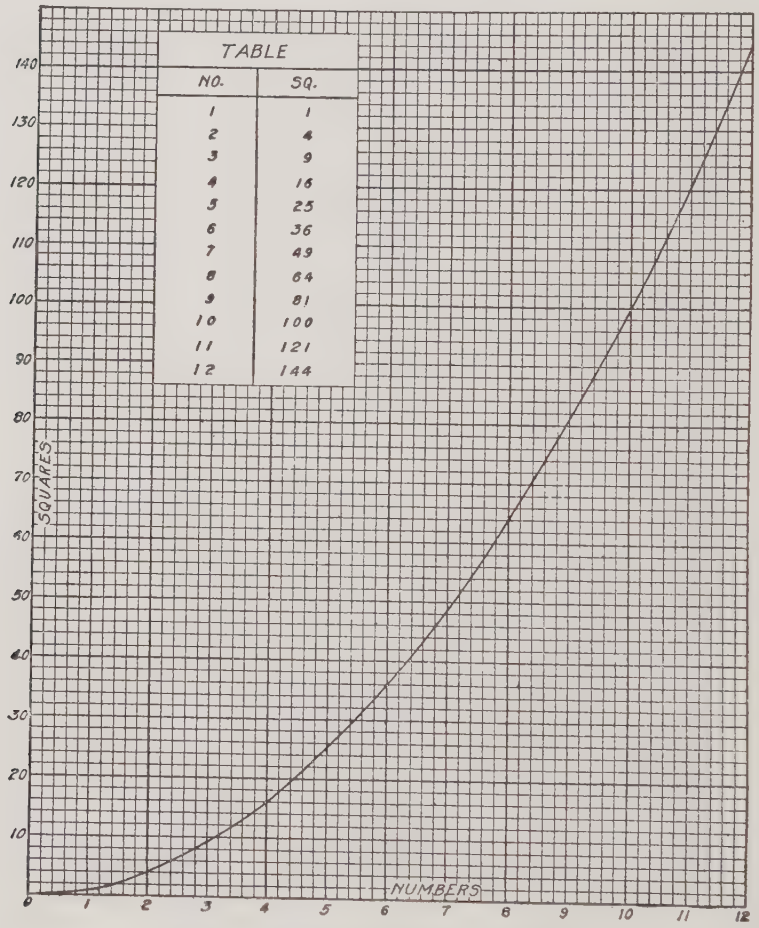


Fig. 58. Curve of Squares and Square Roots

ILLUSTRATIVE CURVES

1. **Curves for Squares and Square Roots.** In Fig. 58 is shown a curve expressing the relation of numbers and the squares of those numbers. Curves of this kind are very practical, and, as can be

readily seen, may be used to save time in all kinds of mathematical work. Compare, for example, the time required to find the *square* of 9.8 mathematically and the time to take the result directly from the above curve. To understand thoroughly the method of using such a curve, the following explanation will be helpful. Five divisions on the horizontal axis equal one unit and, therefore, each divi-

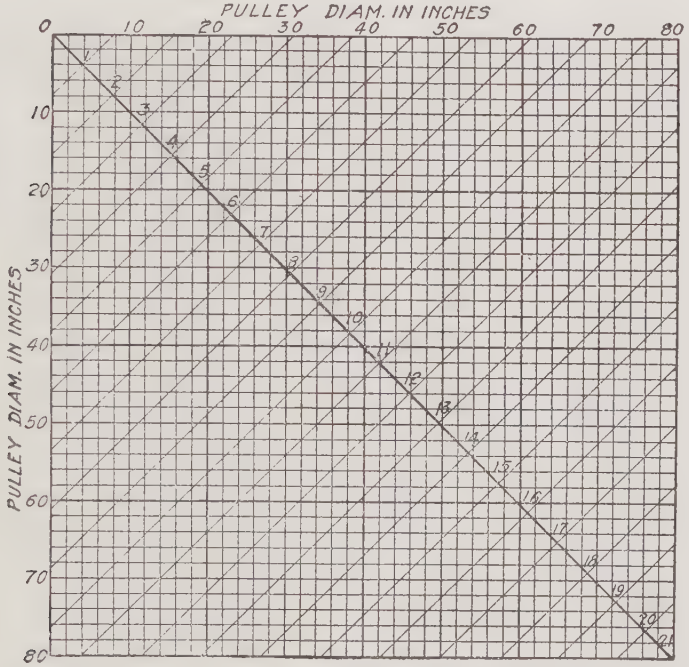


Fig. 59. Curve for Calculating Belt Lengths

sion equals $\frac{1}{5}$, or $\frac{2}{10}$, or .2. On the vertical axis there are five divisions to ten numbers and, therefore, each division equals $\frac{1}{5}$ of 10, or 2. First find on the horizontal axis the point whose value is 9.8. It will be between 9 and 10, just one line short of division 10. Follow this line upward until it meets the curve and from this point go horizontally to the left until the vertical axis is reached. On this line read the value. It is between 90 and 100, three divisions above 90. Each division represents 2, therefore three divisions equals 6. This must be added to 90, giving as a final result 96 approximately.

To find the *square root* of a number the above method would be reversed, using the same curve. Find the number on the vertical axis whose root is to be extracted, then follow horizontally across the sheet to the point of intersection with the curve and then downward to the horizontal axis. The value found there will be the square root; for instance, the square root of $96 = 9.8$ approximately.

2. **Curve for Calculating Length of Belt.** A very useful curve, Fig. 59, may be plotted by which the length of belt required for any sized pulleys between two shafts at any distance can be found at a moment's notice. Lay off pulley diameters in inches on both horizontal and vertical axes to the same scale and draw a diagonal line from the origin at an angle of 45 degrees as shown. Dividing this line into exactly twenty-one equal parts and drawing lines at right angles to the diagonal which meet the vertical and horizontal axes, the figure is completed. The method of using this curve is as follows: Assuming 30-inch and 50-inch pulleys, take, say, 30 inches as the vertical value and 50 inches as the horizontal and find their point of intersection. This point falls about half way between divisions 10 and 11. The length of belt wrapping the pulleys is, therefore, 10.5 feet in length. Assuming 20 feet as the distance between shafts, the total length of belt will be twice this distance plus the length wrapping pulleys as found above, or $(2 \times 20) + 10.5 = 50.5$ feet.

In engineering work there are many practical curves that can be made in a similar manner to this belt curve which will be great labor-saving devices where similar computations are being made day after day. It will readily be seen how much shorter time is required to find the length of pulley from the curve than if the mathematical formula was used. By the regular formula

$$L = 3.1416 \frac{(D+d)}{2} + 2l$$

where L is length of belt in inches; l, distance between shafts in inches; D, diameter of large pulley in inches; and d, diameter of small pulley in inches. Substituting the same values as used above, that is, 30 inches and 50 inches,

$$L = 3.1416 \left(\frac{50+30}{2}\right) + (2 \times 20)\ 12$$

$$= 605.6 \text{ inches, or } 50 \text{ feet } 6 \text{ inches approximately.}$$

3. **Gross Income Curve for Steam Railroad.** A certain steam railroad carrying suburban traffic was opened for business in 1898 and received an increasing amount of business up to the year 1911 when there was a drop. The manager of the road was requested by the board of directors to present a record showing the gross income of the road for each year of operation. He presented the curve, Fig. 60. The co-ordinates of the points are given in connection with the curve. Years are laid off on the horizontal axis and incomes on the vertical axis, 5 spaces on the former equaling 1 year and 1 space on the latter equaling $100,000. The first point is located as in the other cases by reading to the right of the origin on

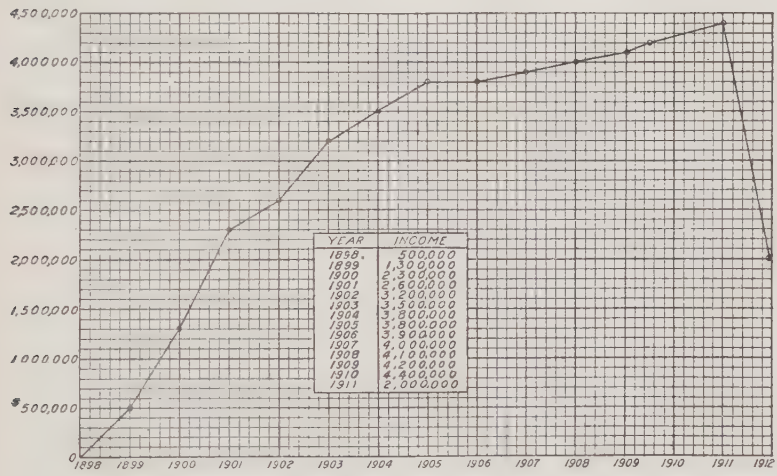


Fig. 60. Steam Railroad Gross Income Curve

the horizontal axis one year; then reading up the vertical line to $500,000; the intersection of these two lines gives the point wanted on the curve. Continuing in like manner for the other points, lines are drawn through the points as noted in Fig. 60. The curve then shows at a glance that the income per year increased each year up to 1911 and here it dropped to $2,000,000. This drop was readily traceable to the influence of a parallel electric line. As this new road made regular trips throughout the day, and much more frequently than the steam road, the electric road received most of the suburban business. This curve shows the relative value of the yearly income much more clearly and strikingly than many pages of statistics.

4. **Contract Price for Sewer Pipe.** A very interesting and practical example of the use of curves in Civil Engineering estimating is shown in Fig. 61. Making use of the data obtained from a

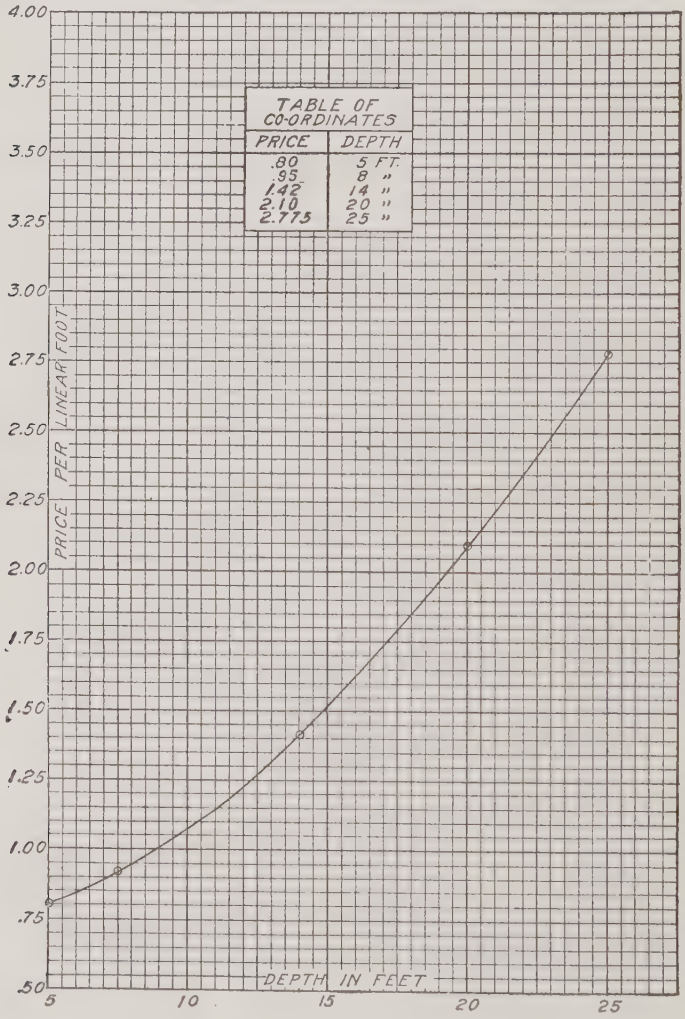


Fig. 61. Curve for Contract Price of Twelve-Inch Sewer Pipe

large number of estimates of sewer work, an engineer has arranged the values in table form and drawn the curve as shown. By means of this curve, the estimator can tell at a glance without further cal-

culation what will be the contract price per foot to bid on 12-inch sewer pipe.

The student should plot this curve from the co-ordinates given in the table, taking for the vertical value the price per linear foot

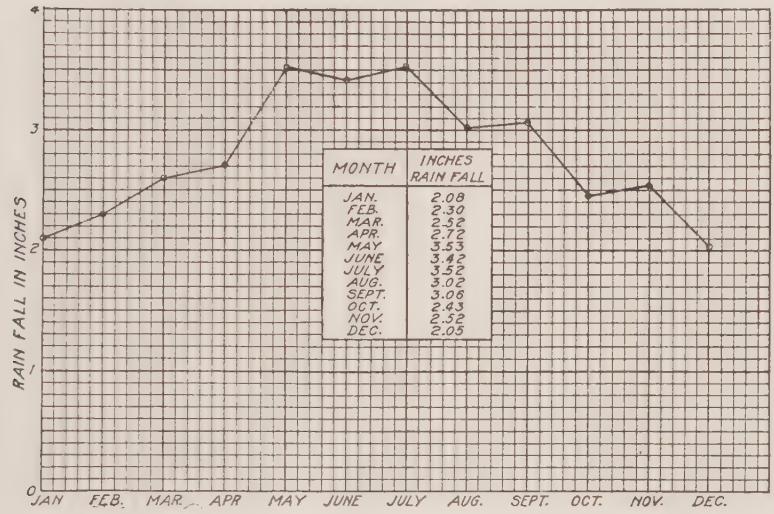


Fig. 62. Mean Rainfall Curve

of this 12-inch sewer and for the horizontal value the depth of excavation of the sewer. This curve applies only to a vitrified clay tile sewer 12 inches in diameter, but in practice curves could easily be drawn for every possible case.

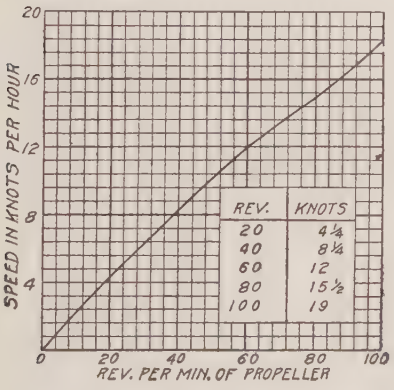
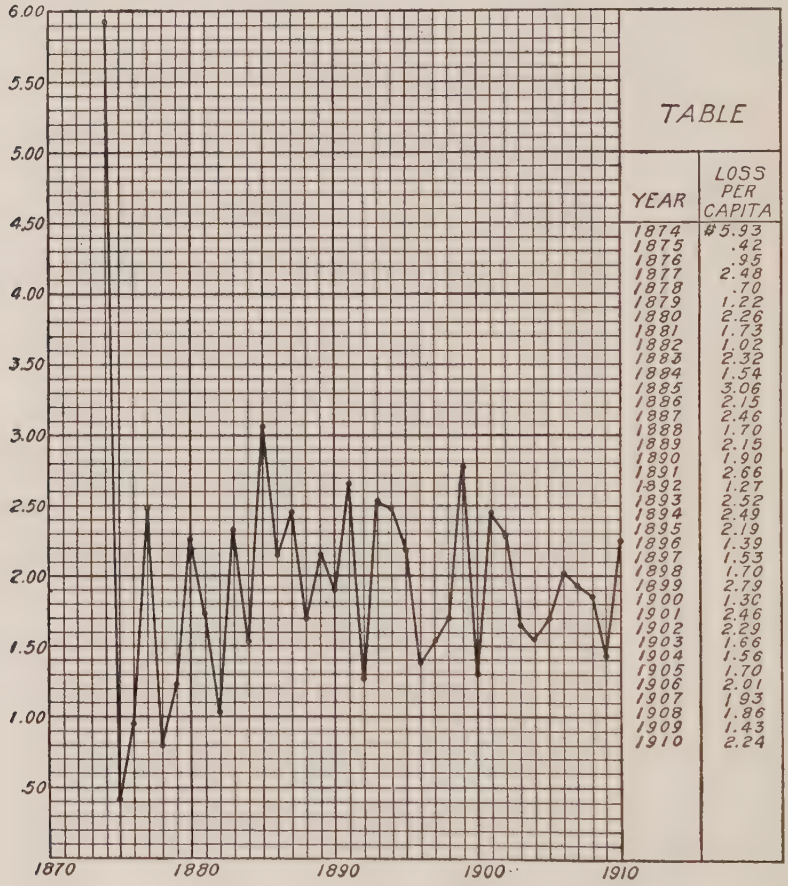


Fig. 63. Speed Curve for Steamboat

5. **Mean Precipitation Curve.** The student will notice in this curve, Fig. 62, that the months of the year are plotted to inches rainfall for each month. On the horizontal axis lay off the months to the right beginning with January at the origin, and letting one month equal five spaces. On the vertical axis lay off the rainfall in inches, using a scale of ten spaces for every inch. The curve shows at a glance that the precipitation is least in January and great-

est from May to July. The student will also notice that the rainfall increases almost gradually from January to April and from this point jumps to a high value in May, remaining about the same



TABLE

YEAR	LOSS PER CAPITA
1874	$5.93
1875	.42
1876	.95
1877	2.48
1878	.70
1879	1.22
1880	2.26
1881	1.73
1882	1.02
1883	2.32
1884	1.54
1885	3.06
1886	2.15
1887	2.46
1888	1.70
1889	2.15
1890	1.90
1891	2.66
1892	1.27
1893	2.52
1894	2.49
1895	2.19
1896	1.39
1897	1.53
1898	1.70
1899	2.79
1900	1.30
1901	2.46
1902	2.29
1903	1.66
1904	1.56
1905	1.70
1906	2.01
1907	1.93
1908	1.86
1909	1.43
1910	2.24

Fig. 64. Curve for Fire Loss per Capita

through June and July, and gradually dropping to its lowest value again in January.

6. **Speed Curve for Steamboat.** The curve shown in Fig. 63 is an actual plot of the data found in the speed trial of a steamboat. A speeder-counter was used to obtain the number of revolutions per minute of the propeller shaft; the log gave the speed in knots

per hour. Using revolutions of the shaft as horizontal values and knots as vertical values, the curve is plotted. Having the curve plotted for a limited range of speeds, the engineer could easily extend it and be able to take values for the speed at any given number of revolutions.

7. **Curve Showing Fire Loss per Capita.** Statistical curves of the type in Fig. 64 show how much more easily one can grasp the situation from a curve than from a column of figures. It is necessary to study the figures up and down to get a comparative idea, while the curve shows at a glance when the loss was great and when small. Other similar curves could easily be made up covering all sorts of tabulated data.

8. **Sunday Current Load Curve.** The current values in amperes delivered by the generators of a trolley system have been recorded in the power house for every period of the day by a proper recording instrument. These two quantities, time and amperes of current, being known, a curve may now be plotted showing the variations of current consumption throughout the day. This curve, Fig. 65, is an exact copy of one which is on the desk of the manager every morning. The managers of these large companies need to have their information systematized and every bit of data as to the operation of the road must be boiled down to its simplest form. This curve shows at a glance at what time the maximum load came on the line, the value of the load, and the variations during the day. Without this curve it would require, perhaps, hours for him to get the information from the data, while with the curve a five minutes' study will give him a clear conception of the whole situation.

Time	Load in Amperes
12:00 P.M.	37,500
4:00 A.M.	12,500
9:00 A.M.	28,000
1:00 P.M.	45,000
3:00 P.M.	78,000
4:00 P.M.	75,000
5:00 P.M.	80,000
6:00 P.M.	74,000
8:00 P.M.	78,500
12:00 P.M.	48,000

The student can go deeper into the meaning of this curve and find the reason for the heavy load between one o'clock in the afternoon and midnight.

9. **Locomotive Repair Curve.** Sometimes it happens that for sake of comparison two curves are plotted on the same sheet. In the case of the curve *A*, Fig. 66, there is a reason for the drop in

the annual cost of repairs per locomotive, which can be made clear by the use of curve *B*, showing the increased efficiency of the employes in the shop in connection with this work.

Year	Per Cent Efficiency of Labor	Annual Repair Cost for Locomotive
1905	55	$2308
1906	82	1677
1907	92	1563
1908	87	1683
1909	93	1243

By plotting both curves on one sheet, one can see at a glance that the cost of repairs per locomotive falls as the efficiency of labor increases. The time in years over which the costs and efficiencies were taken is plotted on the horizontal axis. For the sake of clearness in reading the curves as well as plotting them, the vertical

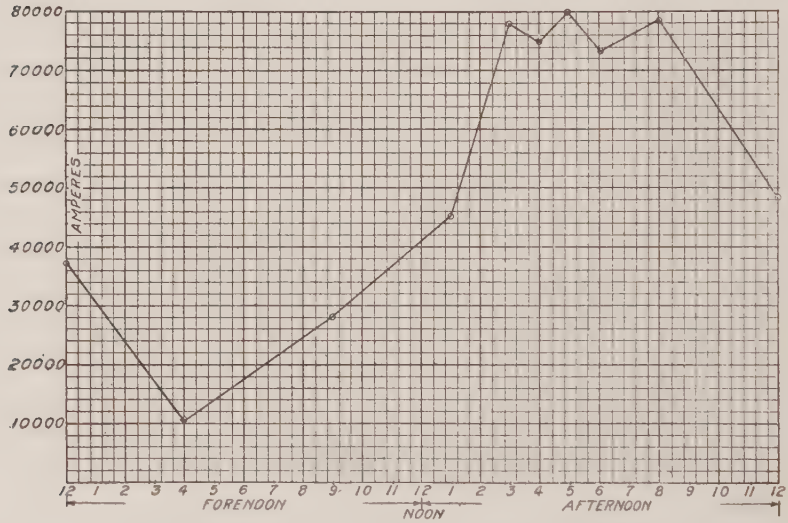


Fig. 65. Sunday Current Consumption Curve for an Electric Road

values for the efficiency curve are indicated on the left and for the cost curve on the right. This system will again be used in the Water and Load diagram, Fig. 67.

10. **Water Power and Load Curves.** In Fig. 67 are shown plotted on the same sheet two curves taken from the power record of a hydro-electric plant. Curve *A* is called the "head" curve, the term *head* referring to the depth of water available to run the water turbines for the generation of power. The *kilowatt* curve shows the number of kilowatts generated at any time during the day or

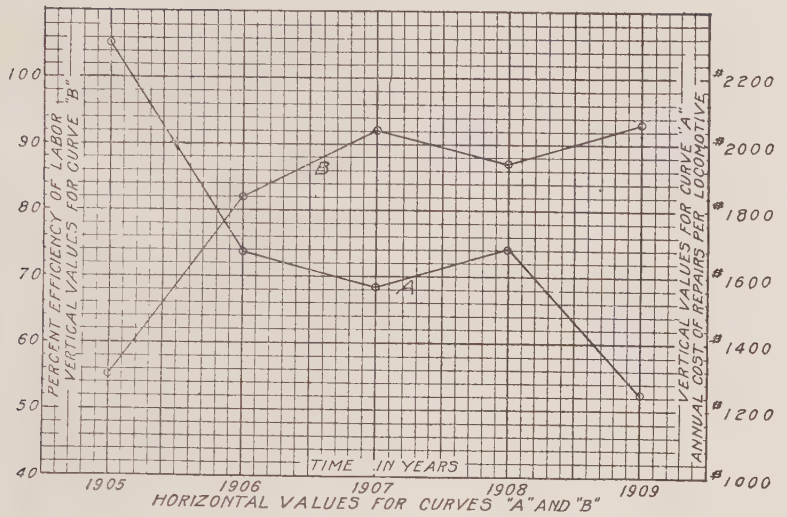


Fig. 66. Locomotive Repair Curves, Showing Cost of Repairs and Efficiency of Labor

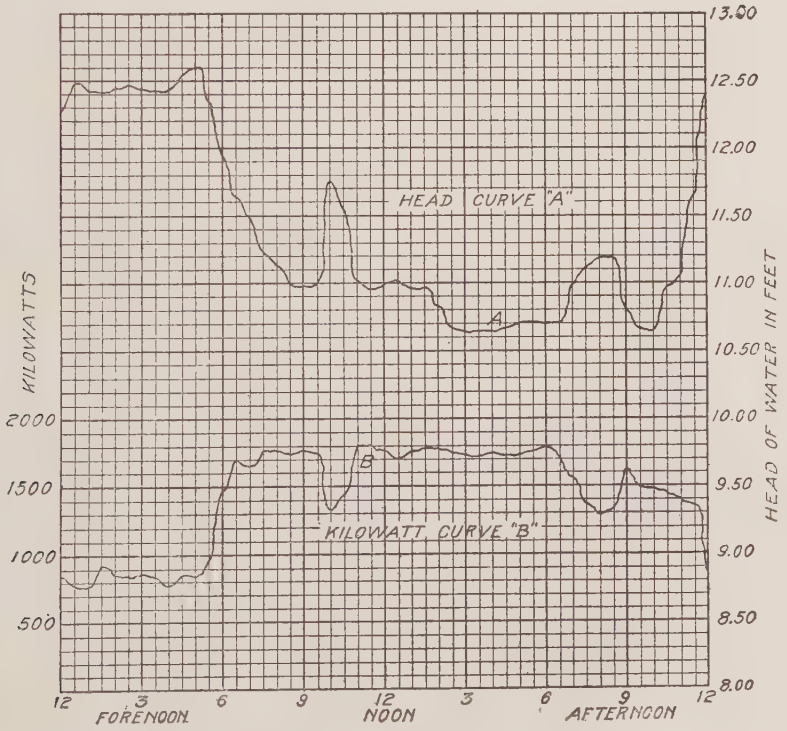


Fig. 67. Power Curves for Hydro-Electric Plant

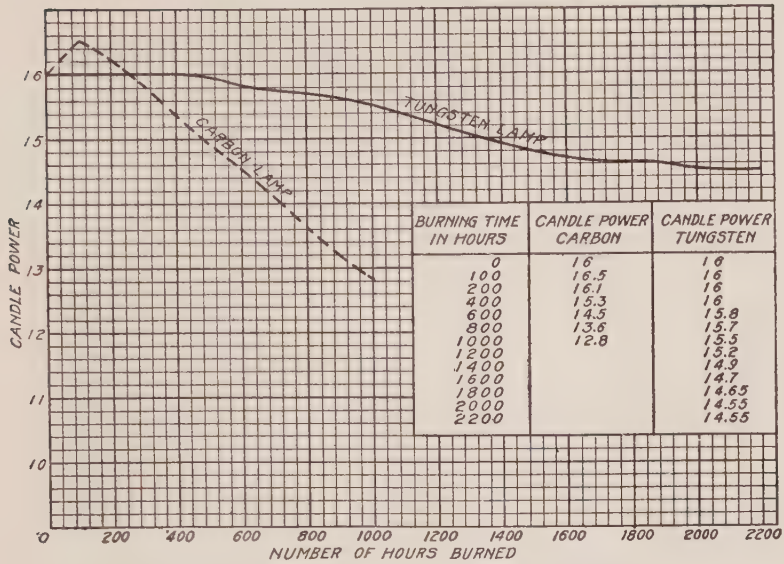
night. The vertical values for the "head" curve are given at the right and for the kilowatt curve at the left.

In finding the meaning of these curves the first point to be noticed is the rise in the kilowatt curve for a drop in the head curve. This indicates that when a load comes on the line, a certain amount of head must be used up in producing this electrical energy, that is, water power has been transformed into electrical energy. The curves are valuable in that they show at a glance the load on the line for any hour of the day and the head of water for any period of the 24 hours. It will be seen that as the load increases or decreases the head changes in the opposite direction.

Time	A. M.		P. M.	
	Head in Feet	Total K. W.	Head in Feet	Total K. W.
12:00	12.29	840		
12:30	12.48	770	11.05	1700
1:00	12.42	770	10.95	1770
1:30	12.41	920	10.95	1790
2:00	12.43	850	10.83	1790
2:30	12.46	830	10.68	1740
3:00	12.43	860	10.65	1730
3:30	12.41	840	10.65	1720
4:00	12.41	770	10.65	1740
4:30	12.58	870	10.66	1730
5:00	12.63	840	10.70	1730
5:30	12.36	940	10.70	1760
6:00	11.94	1470	10.70	1790
6:30	11.62	1700	10.70	1700
7:00	11.50	1660	11.00	1560
7:30	11.25	1770	11.13	1380
8:00	11.15	1770	11.19	1300
8:30	11.00	1760	11.19	1310
9:00	11.05	1770	10.80	1620
9:30	11.00	1760	10.66	1590
10:00	11.75	1330	10.64	1590
10:30	11.55	1440	10.93	1460
11:00	11.05	1800	11.04	1310
11:30	10.95	1800	11.63	1270
12:00	11.00	1770	12.40	880

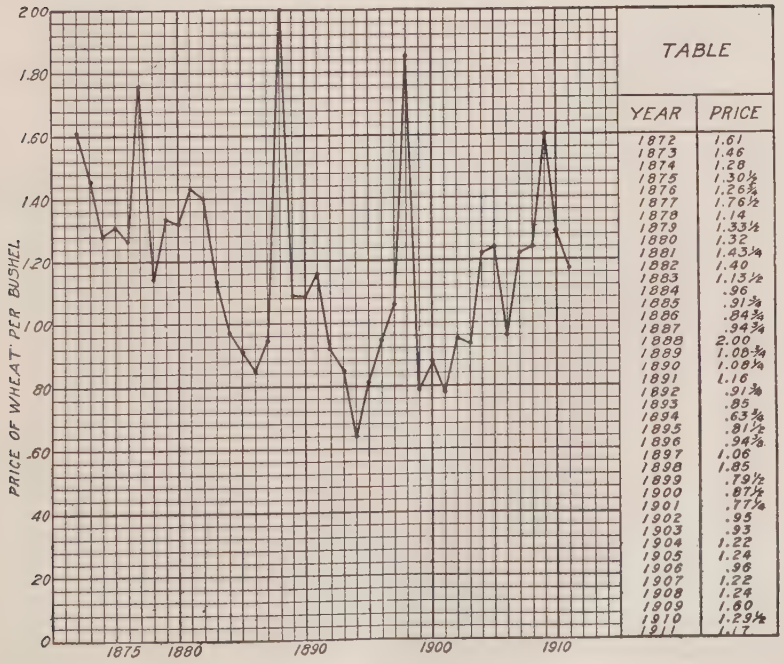
11. **Comparison Curves for Incandescent Lamps.** The two curves shown in Fig. 68 represent actual tests of the efficiency of the old type carbon filament lamps and the new tungsten filament lamps and give a striking graphical picture of the improvement of the new over the old style lamp. Horizontal values represent hours burned, and vertical values candlepower of the lamps. Note that the candlepower scale has been started for convenience at 9, it being evident that if it started at 0 the curves would be higher up but otherwise the same.

The meaning of the curves is clear. The carbon lamp reached the "smashing point," as it is called, in 1,000 hours, whereas the tungsten lamp had only dropped in candlepower to 91 per cent



BURNING TIME IN HOURS	CANDLE POWER CARBON	CANDLE POWER TUNGSTEN
0	16	16
100	16.5	16
200	16.1	16
400	15.3	16
600	14.5	15.8
800	13.6	15.7
1000	12.8	15.5
1200		15.2
1400		14.9
1600		14.7
1800		14.65
2000		14.55
2200		14.55

Fig. 68. Candlepower Curves for Carbon and Tungsten Lamps



TABLE

YEAR	PRICE
1872	1.61
1873	1.46
1874	1.28
1875	1.30½
1876	1.26⅞
1877	1.76½
1878	1.14
1879	1.33¼
1880	1.32
1881	1.43¼
1882	1.40
1883	1.13½
1884	.96
1885	.91⅜
1886	.84⅜
1887	.94¾
1888	2.00
1889	1.08¾
1890	1.08⅛
1891	1.16
1892	.91⅜
1893	.85
1894	.63⅜
1895	.81½
1896	.94⅜
1897	1.06
1898	1.85
1899	.79½
1900	.87½
1901	.77⅛
1902	.95
1903	.93
1904	1.22
1905	1.24
1906	.98
1907	1.22
1908	1.24
1909	1.60
1910	1.29½
1911	1.17

Fig. 69. Curve Showing Variations in Market Price of Wheat

of its original value in 2,200 hours. Further, the tungsten lamp held a uniform candlepower practically for 600 hours, while the carbon lamp began to decline rapidly after 100 hours of burning.

12. **Curve of Wheat Prices.** The price of wheat per bushel varies considerably from year to year, as may be seen from Fig. 69. A fairly good idea of the variation can be obtained by running down the column of figures in the table, but it is not nearly as easy to carry the maximum and minimum prices as to have them pictured

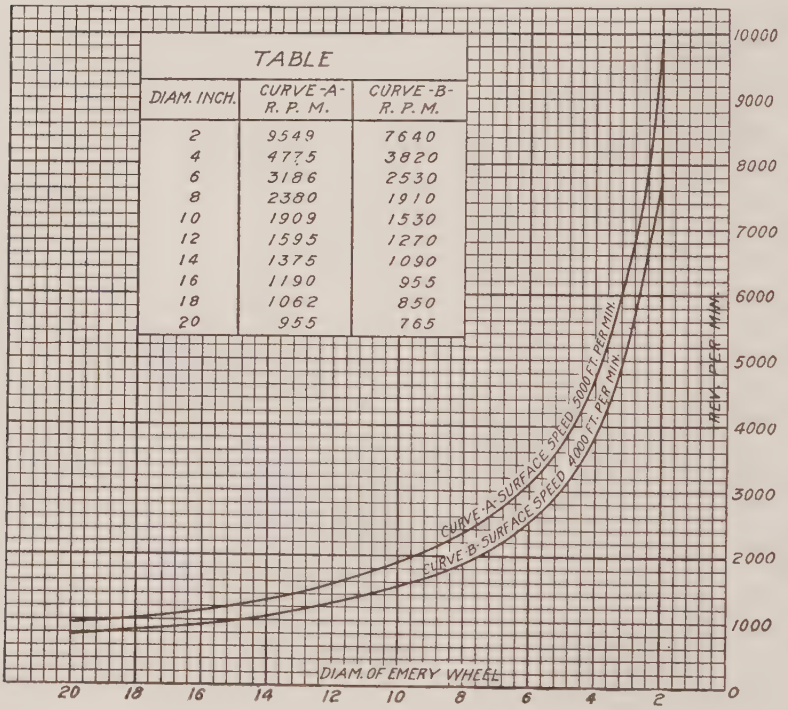


Fig. 70. Speed Curves for Emery Wheels and Pulleys

graphically by a curve. The curve is plotted from the data given in the table and a glance will show that the maximum price is \$2 per bushel in 1888 and the minimum price 63¾c. in 1894. The value of such curves in all business lines where prices of commodities vary from time to time is readily seen.

13. **Speed Curves for Wheels or Pulleys.** The curve, Fig. 70, shows how, for a certain surface or rim speed, the revolutions

per minute of an emery wheel is obtained when its diameter is known. In this particular case curve *A* and curve *B* represent the usual surface speeds of the two grades of emery wheels. Similar curves may be drawn for the usual surface speed of any wheel or pulley and then with the diameter given the revolutions may be obtained, or with the revolutions given the diameter may be obtained. Such a curve is a very practical one and useful in all kinds of machine shops.

CURVE PROBLEMS FOR PRACTICE

1. Plot the curve from the following data for the cost of high-speed compound engines, using thousands of dollars for vertical values and horsepower for horizontal values. What will be the cost of a 750 horsepower engine of this type?

Dollars	Horsepower
6400	400
9600	600
12800	800
16000	1000
24000	1500
32000	2000
40000	2500
48000	3000

2. Plot the curve from the following data for the increase of Chicago's population, using population in thousands as vertical values and years for horizontal values. Assuming the same increase in the next decade as in the last, find the population of Chicago in 1920.

Population	Years
109,206	1860
178,492	1865
306,605	1870
400,500	1875
503,185	1880
665,000	1885
1,099,850	1890
1,366,813	1895
1,698,575	1900
1,949,116	1905
2,195,551	1910

3. Plot the curve from the following data for the areas and diameters of circles, using areas for vertical values and diameters for horizontal values. What is the diameter of a circle having an area of 40 square inches?

Area Sq. Inches	Diameter Inches
.7854	1
3.1416	2
7.0686	3
12.566	4
19.635	5
28.274	6
38.485	7
50.265	8
63.617	9
78.540	10
95.033	11
113.097	12

4. Plot the curve from the following data for the steam consumption in a steam turbine, using pounds of steam per h.p. hour as vertical values, and horsepowers as horizontal values. Find from the curve the steam consumption per h.p. hour of a 450-h.p. turbine.

Pounds Steam	Horsepower
61.75	300
49.25	400
40	500
34.75	600
34	700
38	800

5. Plot the curve from the following data for a centrifugal pump. Plot the values given in the table, using gallons per minute as vertical values, and horsepower per foot of lift as horizontal values. Find from the curve how many gallons of water can be lifted per foot per minute with a horsepower of 25.

Gals. per Min.	Horsepower per Foot Lift
10,000	4.1
20,000	8.
40,000	15.6
80,000	31.4
100,000	39.
120,000	46.75
130,000	51.

6. Plot a smooth curve showing the water consumption per hour for a 400-kilowatt steam turbine by means of the following values. Plot pounds of water per hour as vertical values and horsepower as horizontal values. From the curve determine how many pounds of water per hour will be required when the steam turbine is delivering 700 horsepower.

Lbs. Water per Hr.	Horsepower
5,000	300
7,500	500
11,000	750
13,000	875
17,500	1100

7. Plot the following values for load and efficiency for an air-blast transformer and draw a smooth curve through points plotted. Plot per cent efficiency as vertical values and per cent loads as horizontal values. Determine from the curve at what load the highest efficiency of the transformer may be expected.

Per Cent Load	Per Cent Efficiency
150	98.20
125	98.29
100	98.32
75	98.24
50	97.88
25	96.43
10	92.00
5	85.00
2.4	70.00
1.25	50.00
0	0

APPENDIX

*THE SLIDE RULE

The slide rule is an instrument for working mathematical calculations easily and rapidly, and consists of a grooved base F into which a slide E is fitted, as shown in the illustration. The rule is graduated in a series of four scales A, B, C, and D, and is provided with a runner R which can be moved back and forth to find coinciding points. The starting and ending graduations of each scale are marked 1, and are called the *left* and *right indices*. The graduations A and B on the lower edge of the rule are called the *lower scales*, and C and D on the upper edge, the *upper scales*. A and B are usually used for problems in multiplication, division, and proportion, as their length gives more sub-divisions, and thus more accurate results.

METHOD OF USING THE SLIDE RULE

Proportion. To illustrate the use of the slide rule in solving a problem in proportion, first set the left-hand indices of A and B in coincidence; then each reading on B will bear the same ratio to the coinciding reading on A as the ratio between the indices, viz, a ratio of 1:1. Now if, by moving the slide to the right, 1 on B is made to coincide with 2 on A the readings on B will then have the ratio of 1:2 to the coinciding readings on A. Therefore, to solve a proportion, use the following rule:

Rule. *Set the B scale so that the first term of any proportion on this scale is coincident with the second term on the A scale. Then, without moving the slide, find the third term of the proportion on the B scale, and the coincident number on the A scale will be the unknown fourth term.*

It is important to observe that the values which the several divisions of the scales may have will depend directly upon the value assigned to the left-hand index figure on the scale. On the slide rule shown, this index figure is designated 1. It may, however, be taken to represent any value that is a multiple or sub-multiple of 10. The left-hand index 1 of the A scale may be regarded as 1, 10, 100, 1,000,

*This section is introduced for those who care to use a slide rule. The study of this section and the use of the rule are optional with the student.

etc., or as .1, .01, .001, .0001, etc. When the initial value has been assigned to the index, the ratio of value must then be maintained throughout the whole scale. For example, if 1 on A is taken to represent 10, the main divisions 2, 3, 4, etc., are read as 20, 30, 40, etc. On the other hand, if the fourth main division is read as .004, then the left index figure of the scale is read as .001.

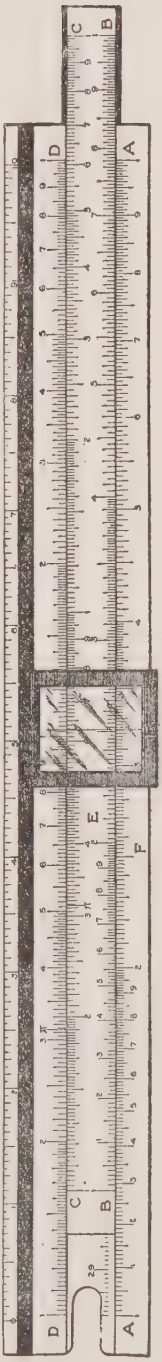
Examples. 1. Find the unknown term in the proportion $10 : 19 : : 2 : x$.

Solution. Take 1 (10) on B, and move the slide to the right until the 1 coincides with 1.9 (19) on A; then under 2 on B, the value 3.8 is found on A, which is, therefore, the unknown term of the proportion. If the index does not fall exactly on a graduation, an estimation must be made.

2. Solve $6 : 18 :: 2 : x$.

Solution. It will be noticed that the scales begin at the left with 1, and not with zero. Therefore, the reduction made in Example 1 is not possible here, as .6 (6) cannot be found on B. If, however, 6 on B is brought into coincidence with 1.8 (18) on A, it will be found that 2 on B is beyond the limits of A. Therefore, proceed as follows: With 6 on B coinciding with 1.8 on A, set the runner so that the hair is over the right index of B. Now push the slide to the right until the left index comes under the hair. Then under 2 on B, the reading 6 will be found on A. 6 is, therefore, the fourth or unknown term of the proportion. In solving examples such as this, the slide may, of course, be moved to the right or to the left, as may be required according to the numbers involved.

Multiplication. Rule. *Move the slide so that the index on the B scale coincides with the multiplicand on the A scale; then the product can be read on the A scale under the multiplier on the B scale.* If preferred, the slide may be moved so that the index on the B scale coincides with the multiplier on the A scale; then the product can be read on the A scale under the multiplicand on the B scale.



The Slide Rule.

Examples. 1. Multiply 25 by 15.

SOLUTION. Move the slide until the left index of *B* is over 2.5 on *A*. Then under 1.5 on *B*, 3.75 will be found on *A*. The desired product is, therefore, 375, as it is clear by an inspection of the problem that the product must contain three digits.

2. Multiply 361 by 119.

SOLUTION. Move the slide so that the index on *B* is over 3.61 on *A*. Under 1.19 on *B*, read 4.29 on *A*. It may be seen by inspection that the product will have five numerical places. The answer is, therefore, 42,900 approximately. In large numbers, such as those used in this example, approximate results only can be obtained, but they are sufficiently accurate for all practical purposes.

Division. Rule. *Move the slide so that the divisor on the B scale will coincide with the dividend on the A scale. The index on the B scale will then coincide with the quotient on the A scale.*

Examples. 1. Divide 256 by 16.

SOLUTION. Move the slide so that 1.6 on *B* coincides with 2.56 on *A*. Coinciding with the left-hand index of *B*, 1.6 will be found on *A*. The quotient is 16.

2. Divide 962 by 13.

SOLUTION. Move the slide until 1.3 on *B* coincides with 9.62 on *A*. Coinciding with the left-hand index on *B*, 7.4 will be found on *A*. The quotient is 74.

Square and Square Root. The upper scale *D* is so related to the lower scale *A* that readings taken on *D* are the squares of the coinciding readings on *A;* while the readings on *A* are, of course, the square roots of the coinciding readings on *D*.

Examples. 1. Find the square of 25.6.

SOLUTION. Move the runner so that the hair coincides with 2.56 on *A*. The hair is then found to coincide with 6.55 approximately on *D*. The square is, therefore, 655.

2. Extract the square root of 311.

SOLUTION. Move the runner to 3.11 on *D*. The hair will then be found to coincide with 1.76 on *A*. 17.6 is, therefore, the square root required.

Cube. Rule. *To find the cube of a number, move the slide until the right or left index of the B scale coincides with the given number as located on the A scale. Then over identically the same number on the C scale find the cube of the number on the D scale.*

Example. Find the cube of 9.3.

SOLUTION. Move the slide until the right index of B coincides with 9.3 on A. The reading 9.3 on C will then coincide with 8.04 on D. The desired cube is, therefore, 804.

It is possible by means of the slide rule to obtain the cube root and higher roots of numbers, but the process is so involved that the use of logarithms is recommended.

Trigonometric Problems. If the slide E is completely withdrawn, three scales will be found on the reverse side. S, the upper one, is a scale of natural sines; T, the lower one, is a scale of natural tangents, and L, the middle one, is a scale of equal parts giving the logarithms corresponding to the numbers on A. When sines or tangents of angles are desired place the slide in the groove with this side up, and with the right and left indices coinciding.

Rules. *The angles will be found on S and their corresponding sines will be found on D. Those found on the left half of D must be divided by 100; while those on the right half must be divided by 10.* For example, $\sin 2° 30' = 0.0436$; $\sin 35° 30' = 0.581$.

To find the tangent of any angle, find the angle on T. The tangent will then be the coincident reading on A, divided by 10. For example, $\tan 21° = 0.384$. Tangents from $5° 43'$ to $45°$ only, are given on the scale. The tangent and sine are approximately equal for angles less than $5° 43'$. For angles larger than $45°$, use the formula

$$\tan A = \frac{1}{\tan (90° - A)}$$

Logarithms. The method of finding the logarithms of numbers is as follows:

Rule. *With the slide in its original position, move it so that the left index of B coincides with the number on A of which the logarithm is to be taken. Reversing the entire slide rule, the logarithms may be read on the middle scale L, the number coinciding with the index in the notch on the base of the rule.* For example, to find the logarithm of 5, move the slide to the right until the index on B coincides with 5 on A. Without moving the slide, reverse the slide rule and the reading on L is found to be 7. The logarithm is, therefore, .700.

Inverting the Slide. When the slide is removed and turned end for end, so as to bring the B scale adjacent to the D scale, and is moved

until the indices coincide, the readings on B are the reciprocals of the coinciding readings on A, and *vice versâ*. This is useful in inverse proportion.

Inverse Proportion. When in a problem *more* requires *less* or *less* requires *more*, an inverse proportion must be formed. This proportion may easily be solved by inverting the slide, when it will be found that the coinciding divisions of the scales, A and B, form a series of inverse ratios. By means of the runner, the readings may be readily taken off.

Examples. 1. If 12 carpenters build a house in 48 days, how long would it take 16 carpenters to build the same house?

SOLUTION. Move the slide inverted to the left until 1.2 on B coincides by means of the runner with 4.8 on A. Then 1.6 on B will be found, by means of the runner, to coincide with 3.6 on A. The 16 carpenters will take 36 days to build the house.

2. A train running 20 miles per hour covers a certain distance in 7 hours. How long will it take a train running at 35 miles per hour to cover the same distance?

SOLUTION. Move the slide to the right until 2.0 on B coincides with 7 on A. Then 3.5 on B will coincide with 4 on A. The faster train will accomplish the run in 4 hours.

Location of Decimal Points. In all of the problems so far used in this article, the number of digits in the various products may be readily noticed by inspection, and the decimal point located. For example, multiplying 25 by 15 gives three digits in the result. If the number of digits in the result is not readily noticed by inspection, the following method will be found useful.

Multiplication. When the slide projects to the left, the number of digits in a product equals the sum of the digits in the multiplier and multiplicand, and when the slide projects to the right, the number of digits in the product equals the sum of the digits in the multiplier and multiplicand less 1.

Example. $1.5 \times 17 = 25.5$.

SOLUTION. The sum of the digits of the two factors is 3, and the slide projects to the right. The product will, therefore, contain $3 - 1$ or 2 digits.

Division. The number of digits in a quotient when the slide projects to the left equals the number of digits in the divisor subtracted from the number of digits in the dividend, and when the slide

projects to the right equals the number of digits in the divisor subtracted from the number of digits in the dividend plus 1.

Example. $22.5 \div 15 = 1.5$.

SOLUTION. Subtracting the number of digits in the divisor from the number of digits in the dividend gives zero. Since the slide projects to the right, this difference is increased by 1, giving one digit in the quotient.

A little practice with problems of this sort will make it a simple matter to correctly determine the position of the decimal point.

A perusal of the article on the slide rule may have impressed the reader with the fact that it is a rather complicated instrument. Its value depends in a large measure upon the quickness with which the operator can assign the correct value to each graduation of the different scales. There are, of course, certain limitations to its use on account of the size of the scales, which limitations can only be discovered by repeated trials and the results in many cases, especially where the numbers are large, can only be approximated. It must, therefore, be left to the student himself to determine the extent to which he will develop an ability to use the instrument, bearing in mind that many engineers have found it exceedingly useful in their work.

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS*

Deg.	Sin.	Cos.	Tan.	Cot.	Deg.	Sin.	Cos.	Tan.	Cot.
0	.00000	1.0000	.00000	∞	46	.71934	.69466	1.0355	.96569
1	.01745	.99985	.01746	57.290	47	.73135	.68200	1.0724	.93252
2	.03490	.99939	.03492	28.636	48	.74314	.66913	1.1106	.90040
3	.05234	.99863	.05241	19.081	49	.75471	.65606	1.1504	.86929
4	.06976	.99756	.06993	14.301	50	.76604	.64279	1.1918	.83910
5	.08716	.99619	.08749	11.430	51	.77715	.62932	1.2349	.80978
6	.10453	.99452	.10510	9.5144	52	.78801	.61566	1.2799	.78129
7	.12187	.99255	.12278	8.1443	53	.79864	.60182	1.3270	.75355
8	.13917	.99027	.14054	7.1154	54	.80902	.58779	1.3764	.72654
9	.15643	.98769	.15838	6.3138	55	.81915	.57358	1.4281	.70021
10	.17365	.98481	.17633	5.6713	56	.82904	.55919	1.4826	.67451
11	.19081	.98163	.19438	5.1446	57	.83867	.54464	1.5399	.64941
12	.20791	.97815	.21256	4.7046	58	.84805	.52992	1.6003	.62487
13	.22495	.97437	.23087	4.3315	59	.85717	.51504	1.6643	.60086
14	.24192	.97030	.24933	4.0108	60	.86603	.50000	1.7321	.57735
15	.25882	.96593	.26795	3.7321	61	.87462	.48481	1.8040	.55431
16	.27564	.96126	.28675	3.4874	62	.88295	.46947	1.8807	.53171
17	.29237	.95630	.30573	3.2709	63	.89101	.45399	1.9626	.50953
18	.30902	.95106	.32492	3.0777	64	.89879	.43837	2.0503	.48773
19	.32557	.94552	.34433	2.9042	65	.90631	.42262	2.1445	.46631
20	.34202	.93969	.36397	2.7475	66	.91355	.40674	2.2460	.44523
21	.35837	.93358	.38386	2.6051	67	.92050	.39073	2.3559	.42447
22	.37461	.92718	.40403	2.4751	68	.92718	.37461	2.4751	.40403
23	.39073	.92050	.42447	2.3559	69	.93358	.35837	2.6051	.38386
24	.40674	.91355	.44523	2.2460	70	.93969	.34202	2.7475	.36397
25	.42262	.90631	.46631	2.1445	71	.94552	.32557	2.9042	.34433
26	.43837	.89879	.48773	2.0503	72	.95106	.30902	3.0777	.32492
27	.45399	.89101	.50953	1.9626	73	.95630	.29237	3.2709	.30573
28	.46947	.88295	.53171	1.8807	74	.96126	.27564	3 4874	.28675
29	.48481	.87462	.55431	1.8040	75	.96593	.25882	3.7321	.26795
30	.50000	.86603	.57735	1.7321	76	.97030	.24192	4.0108	.24933
31	.51504	.85717	.60086	1.6643	77	.97437	.22495	4.3315	.23087
32	.52992	.84805	.62487	1.6003	78	.97815	.20791	4.7046	.21256
33	.54464	.83867	.64941	1.5399	79	.98163	.19081	5.1446	.19438
34	.55919	.82904	.67451	1.4826	80	.98481	.17365	5.6713	.17633
35	.57358	.81915	.70021	1.4281	81	.98769	.15643	6.3138	.15838
36	.58779	.80902	72654	1.3764	82	.99027	.13917	7.1154	.14054
37	.60182	.79864	.75355	1.3270	83	.99255	.12187	8.1443	.12278
38	.61566	.78801	.78129	1.2799	84	.99452	.10453	9.5144	.10510
39	.62932	.77715	.80978	1.2349	85	.99619	.08716	11.430	.08749
40	.64279	.76604	.83910	1.1918	86	.99756	.06976	14.301	.06993
41	.65606	.75471	.86929	1.1504	87	.99863	.05234	19.081	.05241
42	.66913	.74314	.90040	1.1106	88	.99939	.03490	28.636	.03492
43	.68200	.73135	.93252	1.0724	89	.99985	.01745	57.290	.01746
44	.69466	.71934	.96569	1.0355	90	1.00000	.00000	∞	.00000
45	.70711	.70711	1.0000	1.0000					

*For the purposes of our study in Practical Mathematics, the above table of values for sine, cosine, tangent, and cotangent of angles to degrees only is sufficient. Tables giving values to minutes and seconds are used in more accurate work.

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3

PRACTICAL MATHEMATICS

PART IV

Read Carefully: Place your name and full address at the head of the paper. Any cheap, light paper like the sample previously sent you may be used. Do not crowd your work, but arrange it neatly and legibly. *Study the textbook thoroughly before starting the Examination and do not use the Examination as a guide for study. Do not copy the answers from the Instruction Paper; use your own words, so that we may be sure you understand the subject.*

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1. What is the area of a circle 14 inches in diameter? Of a circle having a radius of 9 inches?

2. The base of a right triangle is 18 feet and the altitude is 24 feet. What is the length of the hypotenuse?

3. The hypotenuse of a right triangle is 16 feet and the base is 9 feet. Find the altitude.

4. The perimeter of a regular hexagon is 90 feet. Find the perpendicular distance from the side to the center of the hexagon.

5. The diameter of the base of a conical roof is 40 feet and the altitude is 15 feet. What is the slant height of this roof?

6. Find the logarithm of 398.

7. Find the logarithm of .0062.

8. Find the number whose logarithm is 2.4472.

9. Find the cube root of 638 by logarithms.

10. Find the weight of a cylindrical granite column 22 feet high and 2 feet 6 inches in diameter. (Granite weighs about 156 pounds per cubic foot.)

11. A set of specifications requires that a hole 100 feet square and 20 feet deep be blasted in solid rock. If the cost per cubic yard for blasting and labor is $3.00, how much will the excavation cost? How many days will be required to plaster the four walls if 100 square yards can be placed in 1 day of 8 hours?

12. The altitude of a certain conical roof is 20 feet, and the diameter of the circular base is 60 feet. Find the square feet of tarred felt required to cover the roof, neglecting laps at seams.

13. A contractor agrees to move a pyramidal marble monument, the area of whose base is 40 square feet and whose height is 60 feet. If the marble weighs 163 pounds per cubic foot, and the contractor receives $3.00 per thousand pounds for the moving, how much money does the contractor receive?

14. A cylindrical standpipe is 100 feet high and 20 feet in diameter. What is its capacity in gallons?

15. A railroad cut has a trapezoidal cross-section. The depth of the cut is 11 feet, the top is 40 feet wide, and the bottom is 25 feet wide. If this cut is 200 feet long, how many cubic yards of earth have been removed?

16. A certain field has the shape of a right triangle, whose hypotenuse is 100 rods; the altitude and the base are 60 and 80 rods, respectively. Find the area.

17. In a right triangle, whose sides are a and b and whose hypotenuse is c, what relations of a, b, and c represent the sine, the cosine, and the tangent of the angle formed by the sides b and c.

18. The perimeter of the base of a regular hexagonal prism is 60 feet; the height is 20 feet. Find the volume. Find the area of one lateral face.

19. Plot the curve from the following data for cubes and cube roots, using cubes for vertical values and numbers 1 to 10 for horizontal values. Find from the curve the cube of 8.4 and check by calculation.

Cubes	Numbers
1	1
8	2
27	3
64	4
125	5
216	6
343	7
512	8
729	9
1000	10

20. From the following table plot a curve showing how the fall of the water surface in a cylindrical tank lying on its side varies with the number of gallons discharged.

Vertical Value	Horizontal Value	Vertical Value	Horizontal Value
0 inches	0 gals.	18 inches	265 gals.
1 inches	4 gals.	20 inches	300 gals.
2 inches	10 gals.	24 inches	375 gals.
3 inches	18 gals.	26 inches	414 gals.
4 inches	28 gals.	30 inches	470 gals.
6 inches	55 gals.	32 inches	498 gals.
8 inches	84 gals.	34 inches	520 gals.
12 inches	147 gals.	35 inches	526 gals.
14 inches	185 gals.	36 inches	530 gals.

After completing the work, add and sign the following statement:
I hereby certify that the above work is entirely my own.
(Signed)

1-4-22

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